Dynamical Systems

Fall 2007 **Dr. D. Lenz**

Homework 6 **Due October 12, 2007**

(1) Consider the linear system $x' = Ax$ with

$$A = \begin{pmatrix} 1 & \alpha \\ 1 & -1 \end{pmatrix},$$

where α is a real parameter.

(a) Determine the flow and sketch a phase portrait for $\alpha = 0, -1, -2$.

(b) How does the system behave for other values of α?

(2) Recall that there are three different types of orbits for an autonomous flow. Consider now the linear differential equation $x' = Ax$ with $A \in \mathcal{L}(\mathbb{R}^n)$ and the associated flow φ.

(a) Which types of orbits can be present for φ if $n = 1$?

(b) Which types of orbits can be present for φ if $n = 2$?

(c) Which types of orbits can be present for φ if $n = 3$?

(3) Solve the differential equation

$$x' = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} t \\ 1 \end{pmatrix}, \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(4) Give an example of $A \in \mathcal{L}(\mathbb{R}^n)$ and $r : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ continuously differentiable with $r(0) = 0$ and $Dr(0) = 0$ such that the origin is stable for $x' = Ax$ and is not stable for $x' = Ax + r(x)$.