
Dynamical Systems

Fall 2007

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Homework 8

Due November 12, 2007

(1) Consider the system

$$y_1' = -y_1, \quad y_2' = -y_2 + z^2, \quad z' = z.$$

- (a) Solve the system.
- (b) Determine the conjugacy H between the system and the linearized system by successive approximation.
- (c) Use H to determine the stable and unstable manifold.

(2) Consider the system

$$y' = -y, \quad z_1' = z_1, \quad z_2' = z_2 + y^2 + yz_1$$

and proceed as in assignment (1).

(3) Consider the system

$$y_1' = -y_1, \quad y_2' = -y_2 + y_1^2 z, \quad z' = z$$

and show that the approximations for Φ in $H = (\Phi, \Psi)$ do not converge globally.

(4) Consider the system

$$z_1' = 2z_1, \quad z_2' = 4z_2 + z_1^2.$$

Show that the approximations for $H = \Psi$ do not converge globally. Furthermore show that if H is twice continuously differentiable then the Jacobian $J(z) = \det DH(z)$ vanishes at $z = 0$ and conclude that the inverse of H is not differentiable at $z = 0$.