(1) Consider the system
\[ y'_1 = -y_1, \quad y'_2 = -y_2 + z^2, \quad z' = z. \]

(a) Solve the system.
(b) Determine the conjugacy \( H \) between the system and the linearized system by successive approximation.
(c) Use \( H \) to determine the stable and unstable manifold.

(2) Consider the system
\[ y' = -y, \quad z'_1 = z_1, \quad z'_2 = z_2 + y^2 + yz_1 \]
and proceed as in assignment (1).

(3) Consider the system
\[ y'_1 = -y_1, \quad y_2 = -y_2 + y_1^2 z, \quad z' = z \]
and show that the approximations for \( \Phi \) in \( H = (\Phi, \Psi) \) do not converge globally.

(4) Consider the system
\[ z'_1 = 2z_1, \quad z'_2 = 4z_2 + z_1^2. \]
Show that the approximations for \( H = \Psi \) do not converge globally. Furthermore show that if \( H \) is twice continuously differentiable then the Jacobian \( J(z) = \det DH(z) \) vanishes at \( z = 0 \) and conclude that the inverse of \( H \) is not differentiable at \( z = 0 \).