Dynamical Systems

Fall 2007 **Dr. D. Lenz**

Homework 9 **Due December 07, 2007**

(1) Show that any solution of

$$x' = -y - x^3,\ y' = x - y^3,$$

converges for $t \to \infty$ towards the trivial fixed point at the origin.

(2) Let $E \subset \mathbb{R}^N$ be open and $F : E \longrightarrow \mathbb{R}$ be two times continuously differentiable and consider the gradient system $x' = -\nabla F(x)$. Show the following

(a) Any limit set of the system consists of critical points only.

(b) If F attains a strict relative minimum in x_0 with $\nabla F(x) \neq 0$ for all points in a neighbourhood of x_0, then x_0 is an asymptotically stable critical point.

(3) Let $F : \mathbb{R}^N \longrightarrow \mathbb{R}^N$ be continuously differentiable and consider the system $x' = F(x)$. Let $V : \mathbb{R}^N \longrightarrow \mathbb{R}$ be continuously differentiable with a strict minimum in 0, $V(x) \to \infty$ for $x \to \infty$ and negative orbital derivative $V^\circ(x)$ for all $x \neq 0$. Show, that the critical point 0 is globally asymptotically stable i.e. it is asymptotically stable and the whole $\mathbb{R}^N$ is basin of attraction.