
Dynamical Systems

Fall 2007

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Midterm

Due November 16, 2007

- (1) (a) Let A, B be real $n \times n$ matrices such that the flows e^{tA} and e^{tB} are linear conjugate (i.e. the topological conjugacy is a linear invertible map). What can you say about A and B ?
- (b) Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be such that for any two solutions u, v of $x' = f(x)$ and $\lambda \in \mathbb{R}$, the function $u + \lambda v$ is a solution as well. Show that f is a linear map.
- (2) Analyse the system (i.e. find fixed points, nullclines etc....)

$$x' = x^2 - y, \quad y' = x - y$$

and draw a phase portrait.

- (3) Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be locally Lipschitz continuous with associated flow φ . Let $M \subset \mathbb{R}^n$ be closed. Show the following:
- (a) M is positively invariant under φ if and only if for any x in the boundary of M there exists an $\delta > 0$ with $\varphi_s(x) \in M$ for all $s \in [0, \delta)$.
- (b) M is invariant if and only if the boundary of M is invariant. (Hint: Part a) is not needed to solve b.)
- (4) Consider the system

$$x' = -7x + 9y, \quad y' = -6x + 8y + y^2$$

and determine a quadratic approximation to the stable manifold at the origin.

- (5) Let $E := \{(x, y) \in \mathbb{R}^2 : x > -1\}$. For $\alpha, \beta \in \mathbb{R}$ let $f : E \rightarrow \mathbb{R}^2$ be given as
- $$(f_1(x, y), f_2(x, y)) = (\sin(\alpha x + \beta xy) - e^y + 1, \log(1+x) + \tanh(\beta y - \alpha xy)).$$
- For which values of α, β is the origin an asymptotically stable fixed point of $(x, y)' = f(x, y)$?