Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have two hours. Show all your work for a full credit. If more space is needed, use the back pages.

Print name: ________________________________

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam, and have observed the time limit given. I started working on this exam at __ : __ and finished at __ : __ on the __th day of October.

Signature: ________________________________

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(1) Find the equation of the line through the origin which perpendicularly meets the line
\[ x = -1 + t, \; y = -2 + t, \; z = -1 + t. \]

(2) What is the volume of the parallelopiped with sides \(2i + j - k\), \(5i + 3k\), \(i - 2j + k\)?
(3) Describe all unit vectors perpendicular to \((1, 0, 0)\) and \((-2, 0, 0)\).

(4) Find an equation for the plane that passes through \((2, -1, 3)\) and is perpendicular to the line \(x = 1 + 3t, y = -2 - 2t, z = 2 + 4t\).
(5) Describe the intersection of the surfaces \( \phi = \frac{\pi}{4} \) and \( z = 1 \).

(6) Let \( f(x, y) = x \cos x \cos y \) and \( x_o = (0, \pi) \).

   (a) Show that \( f \) is differentiable at \( x_o \).

   (b) Find an equation for the tangent plane to \( f \) at \( x_o \).
(7) Let $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ be three vectors in $\mathbb{R}^3$ originating from the origin.

(a) Show that the set of points on the line segment joining the heads of $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\{ s \mathbf{a} + (1 - s) \mathbf{b} \mid 0 \leq s \leq 1 \}$$

(b) Now, describe the set of points in the triangle whose vertices are the heads of the three vectors.
(8) Two surfaces are said to be *tangent* to each other at \( x_0 \) if they intersect at \( x_0 \) and the tangent planes to the two surfaces at \( x_0 \) coincide.

Let \( f(x, y, z) = x^2 + y^2 - z \) and \( g(x, y, z) = z + x^2 + y^2 - xy^3 \). Show that the two level surfaces \( f = 0 \) and \( g = 0 \) are tangent at \( (0, 0, 0) \).
(9) Find the critical points of the following function, and then determine whether they are local maxima, local minima, or saddle points.

\[ f(x, y) = x^2 + y^2 - xy. \]
(10) Find the absolute maximum and the absolute minimum for the function $f(x, y, z) = x + y - z$ on the ball

$\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$. 