Math 212 Multivariable Calculus - Midterm I
Due 5pm March 3rd, 2003

Instructions: You have 2 hours to complete the exam. This is a closed book, closed notes exam. Use of calculators is not permitted. Show all your work for a full credit.

Print name: ________________________________

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam, and observed the time limit specified above.

Signature: ________________________________

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(1) Let \( l_1 \) be the line defined by \( l_1(t) = (t, t, 1 - t) \).

(a) Find the equation of the line \( l_2 \) through the origin which perpendicularly meets \( l_1 \).

(b) Find an equation of the plane that contains \( l_1 \) and \( l_2 \).

(2) Compute the volume of the parallelopiped spanned by \( i + 3j - k \), \( 2i + 3j \), and \( 2i + 5j + k \).

(3) Let \( S \) be the surface defined by \( \phi = \frac{\pi}{3} \) in spherical coordinates. Find an equation for \( S \) in Cartesian coordinates. That is, express \( \phi = \frac{\pi}{3} \) in terms of \( x \), \( y \), and \( z \).

(4) Let \( f(u, v, w) = (e^{u-w}, \cos(v + u) + \sin(u + v + w)) \) and \( g(x, y) = (e^x, \cos(y - x), e^{-y}) \). Calculate \( f \circ g \) and \( D(f \circ g)(0, 0) \).

(5) Let \( S \) be the surface defined by \( z^2 = 2x^2 + 2y^2 \). Find an equation of the tangent plane to \( S \) at \( (1, 1, 2) \).

(6) Calculate the directional derivative of \( f(x, y, z) = e^x + yz \) at \( x_o = (1, 1, 1) \) in the direction of \( w = i + j \) in two ways as directed below.

(a) Compute \( h(t) = f(x_o + tw) \), and then compute the derivative \( h'(0) \).

(b) Compute the directional derivative using the formula \( \nabla f(1,1,1) \cdot w \).

(7) Let \( D \) be the unit disk \( x^2 + y^2 \leq 1 \). Find the absolute maximum and the absolute minimum of the function \( f(x, y) = x^2 + y^2 - \frac{x}{2} - \frac{y}{2} \) on \( D \).

(8) Consider the vector field \( F = xi - yj \).

(a) Find an equation for a flow line for \( F \) passing through \((1,1)\).

(b) Draw four flow lines for \( F \) passing through \((1,1)\), \((-1,1)\), \((1,-1)\), and \((-1,-1)\) respectively. Mark the direction of the flow line with arrow.

(c) Find the divergence of \( F \) and explain why your answer is consistent with the sketch of flow lines.

(9) Verify the identity \( \nabla \cdot (\nabla \times F) = 0 \) for the vector field \( F(x, y, z) = zyi + xy^2j - z^2xk \).