Math 212 Spring 2006
Exam #1

Instructions: You have 90 minutes to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 6 questions. You must show your work to receive full credit. Be sure to indicate your final answer clearly for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. The exam is due by Wednesday, 4 p.m. Good luck!

1. Let \( P = (2, 3, 1), Q = (2, 2, 2), R = (3, 3, -1) \).
   (a) Find the equation of the plane through \( P, Q \) and \( R \).
   (b) Let \( l \) be the line given by \( x = 2 - t, y = 3t, z = 1 + 2t \). Find the intersection point of the plane and the line.

2. Let \( P \) be the plane given by the equation \( x_1 - 2x_2 + 3x_3 + 4 = 0 \) and let \( Q \) be the plane given by all points of the form

   \[ (2, 4, 1) + \lambda(3, 3, 1) + \mu(2, -1, 1), \lambda, \mu \in \mathbb{R}. \]

Determine whether \( P \) and \( Q \) are parallel or not.

**Note:** Two planes are called parallel if they do not intersect, an equivalent condition is that two planes are parallel if their normal vectors are parallel.

3. Consider the two functions \( f : \mathbb{R} \to \mathbb{R}^3, f(t) = (\cos(t), \sin(t), t), g : \mathbb{R}^3 \to \mathbb{R}, g(x, y, z) = x^2 + y^2 + z^2. \)
   (a) Use the chain rule to compute the derivative of \( g \circ f \) at the point \( t = \pi/4 \).
   (b) View the function \( f \) as describing a path in 3-space. Write an equation for the tangent line to this path at the point \( (0, 1, \pi/2) \).
4. An astronaut is floating in the middle of a nebula in outer space. The gas in this nebula is very hot, and she must decrease the temperature she experiences as quickly as possible. In a rectangular coordinate system centered on her, the temperature of the gas (in degrees Centigrade) is described by the equation

\[ T(x, y, z) = -2x + \sin(x^2)y^2 + 2z + 78. \]

(a) Which direction should the astronaut go? (Note that the astronaut is located at (0, 0, 0)).

(b) Would traveling in the direction of the vector \( \mathbf{v} = (-1, -17, 1) \) increase or decrease the temperature she experiences?

5. Consider the graph of the function

\[ f(x, y) = x^3(y^2 - 1) + (x - y)^3. \]

(a) Find the equation of the tangent plane to the graph at \( P = (1, 2) \).

(b) Find a unit vector which is normal to the graph at \( P \).

6. (a) Consider \( h(x, y) = x^y \). Find the partial derivatives \( \frac{\partial h}{\partial x} \) and \( \frac{\partial h}{\partial y} \).

(b) Use (a) and the Chain Rule to find

\[ \frac{d}{dt} \left( f(t)^{g(t)} \right). \]