

1. Find the domain and range of $f(x) = \tan 3x$ and $g(x) = \sqrt{\ln x}$. Compute $f \circ g$ and $g \circ f$ for those values of x where the compositions are defined. Answer:

Domain(f) = $\{x \in \mathbb{R}, \text{ with } x \neq (2k + 1)\frac{\pi}{6}, \text{ for } k \in \mathbb{Z}\}$

Range(f) = \mathbb{R}

For Domain(g), first $x > 0$ for $\ln x$ to be defined, then $\ln x \geq 0$ for the $\sqrt{\ln x}$ to be defined. The last inequality implies $x \geq 1$. Thus, Domain(g) = $[1, \infty)$.

Range(g) = $[0, \infty)$.

2. Find the equation of the line that passes through (2,-3) and is parallel to the line of equation $3x - 2y = 4$.

Answer: $y = \frac{3}{2}x - 6$.

3. Calculate the following limits:

a)

$$\lim_{t \rightarrow 3} \frac{t^3 - 9t}{t^2 - 9}$$

b)

$$\lim_{x \rightarrow 1} \frac{x - 3}{x - 1}$$

c)

$$\lim_{x \rightarrow 0^-} \frac{x}{x - |x|}$$

d)

$$\lim_{x \rightarrow \infty} \frac{2^x}{3^x}$$

e)

$$\lim_{x \rightarrow 0} \frac{\ln(1 + x^2)}{e^x - \cos x}$$

f)

$$\lim_{x \rightarrow \pi} (x - \pi) \csc x$$

g)

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\ln(1 + x)} \right)$$

h)

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - \sqrt{x^2 - x}$$

i)

$$\lim_{x \rightarrow 1} x^{1/(1-x)}$$

j)

$$\lim_{x \rightarrow \pi/2} (\tan x)(\cos 3x)$$

Answers: a)3, b)does not exist, c)1/2, d)0, e)0, f)-1, g)-1/2, h)1, i) e^{-1} , j)-3

4. Use the intermediate value property to show that the equation $x^3 - 3x^2 = -1$ has a solution in the interval $[0,1]$.

Answer: Consider $f(x) = x^3 - 3x^2 + 1$ and check that $f(0)$ and $f(1)$ have different signs. The IVP implies there is some $c \in [0, 1]$ such that $f(c) = 0$. This is equivalent to $c^3 - 3c^2 = -1$, thus c is a solution of the given equation.

5. Show that the curve $y = x^5 + 2x$ has no horizontal tangents. What is the smallest slope that a line tangent to this curve can have?

Answer: 2.

6. Calculate the first derivative of each of the following functions

a) $f(x) = \ln(e^{2x})$. Answer: 2.

b) $g(x) = \sqrt{x^2 + 3}$. Answer: $\frac{x}{\sqrt{x^2+3}}$.

c) $h(x) = \tan(x^2 + \sqrt{x})$. Answer: $[\sec^2(x^2 + \sqrt{x})](2x + \frac{1}{2\sqrt{x}})$.

d) $l(x) = (\ln x)^x$ (use logarithmic differentiation). Answer: $(\ln x)^x \cdot [\ln(\ln x) + \frac{1}{\ln x}]$

7. Find the maximum possible volume of a right circular cylinder if its total surface area, including top and bottom, is 150π .

Answer: 250π

8. Solve the equation $\ln x - \ln(x - 1) = \ln 2$ Answer: $x = 2$.

9. At what rate is the area of an equilateral triangle increasing if its base is 10cm long and increasing at 0.5 cm/s?

Answer: $\frac{5\sqrt{3}}{2} \text{ cm}^2/\text{s}$.

10. Show that the equation $x \ln x = 3$ has exactly one solution in the interval $[2,4]$. Hint: Use the intermediate values property to show there is at least one solution (as in problem 4) and then show that the function $f(x) = x \ln x - 3$ is increasing on the given interval (show that $f'(x) > 0$ for $x \in [2, 4]$).

11. Classify the critical points of the function $f(x) = \sin x - x \cos x$ defined on $(-5,5)$ as local minimum, local maximum or neither.

Answer: $f(\pi)$ is a local (and global) max, $f(-\pi)$ is a local (and global) min, $f(0)$ is neither.

12. Find the inflection points and critical points of $f(x) = x^3 - 3x^2 - 45x$.

Answer: Critical points for $x = -3, 5$, inflection point for $x = 1$.

13. Sketch the graph of $g(x) = \frac{x^2+x-1}{x-1}$. Hint: Divide the numerator by the denominator and write $f(x) = x + 2 + \frac{1}{x-1}$ to see the graph asymptotes.

14. Calculate the first 4 derivatives of $f(x) = (2x + 1)^{100}$ and make a conjecture about a possible value of the n -th derivative. Answer:

$$f'(x) = 100 \cdot 2(2x + 1)^{99}$$

$$f''(x) = 100 \cdot 99 \cdot 2^2(2x + 1)^{98}$$

$$f^{(3)}(x) = 100 \cdot 99 \cdot 98 \cdot 2^3(2x + 1)^{97}$$

The conjecture part is left to the reader.