Halloween Math Challenge

November 4, 2008

1. Two witches, e^w and w^{1000} hit each other with the Derivative Spell. Which one of the two will prevail?
Answer: e^w. The derivative has no effect on her.

2. x^2 takes a ride in the Diffyroller and comes out as $2x$. If $\sin 2x$ takes 3 consecutive rides it will come out as...
Answer: $-8\cos 2x$.

3. The radius of a round magical pumpkin increases at a rate of 1 cm/s. At what rate is its volume increasing when the radius is 10cm?
Answer: $400\pi\ cm^3/s$.

4. A pirate ship leaves Spookytown and heads east so so that at time t hours it is $S(t) = 55t$ nautical miles away from Spookytown. What is the ships velocity at time $t = 100$ hours?
Answer: The ships sails with with constant velocity, 55 miles/hour.

5. What is the area of the largest rectangular cemetery that may be fenced in using 2400 ft of fence?
Answer: 360000 ft^2.

6. A roller coaster follows the curve $y(t) = 12 + 12\sin(4t)$ for a good part of the ride. What is the rate of fastest descent on this portion?
Answer: 48 length unit/time unit

7. A spider gone crazy runs away from its nest with a velocity of 0.2m/s. Write a function $d(t)$ describing the spider distance from its nest after t *minutes*.
Answer: The spider runs 12m/min, thus $d(t) = 12t$.

8. The secret door code of a local Ghostville tavern is

$$e^{\ln 5} - \log_2 4$$

What number would you push on a standard 10 digit keypad?
Answer: 3.

9. A wizard flying on his broom is

$$y(t) = 30\sqrt[3]{t^2}$$

meters above the ground after t seconds from take off. At what time does the wizard ascend at a rate of 2 m/s?
Answer: $t = 1000$ seconds.

10. A candy maker finds that his customers enjoy his sweets as long as he uses anywhere between

1 and 4 lb of sugar per batch. The satisfaction of his costumers,

$$H(x) = x^2 - 5x + 4$$

depends on the quantity of sugar x. What quantity of sugar keeps the costumers happiest?
Answer: 2.5 lb

11. A spider population starting at 10 individuals doubles every 10 days. Write a function of time describing the number of spiders at time t (in days).
Answer: $f(t) = 102^{t/10}$.

12. Come Halloween season, Rags "R" Us sales sum up to $S(t) = 8000 + 2000e^{t/10}$ dollars in day $t \geq 1$. What is the rate of sales increase in day 10?
Answer: 200e dollars/day

13. A scientist looking for the ultimate Halloween treat finds that two of the variables involved are related by $x = e^{3 \ln y}$. He needs y as a function of x. Can you help?
Answer: $y = \sqrt[3]{x}$.

MIDTERM 2 INFO

Midterm Exam 2: Wed, November 5th, in class. No notes, books or calculators are allowed during the exam. However, you will be allowed to write on both sides of an index card (3"x5") and bring the card with you during the exam. The card has to be handwritten by yourself.

Material covered: Sections 3.1 to 3.8 in the textbook, **except** the last two subsections in 3.8 (Derivatives of Logarithmic Functions, Logarithmic Differentiation). Even if not tested explicitly, the material covered by Chapters 1 and 2 is assumed to be known.

SOME PRACTICE PROBLEMS FOR MIDTERM 2

1. Find the equation of the tangent line to the graph of $f(x) = \tan x$ at the point where $x = \pi/4$.
Answer: $y = 2x + 1 - \pi/4$.

2. The demand equation for a particular toy is $p^2 x = 5000$, where x toys are demanded per month when p dollars is the price per toy. It is expected that in t months, where $t \in [0, 6]$, the price of the toy will be p dollars, where $20p = t^2 + 7t + 100$. What is the anticipated rate of change of the demand with respect to time in 5 months?
Answer: -16.6 toys/month (decreasing by 16.6 toys/month).

3. A piece of wire 10ft long is cut into two pieces. One piece is bent into the shape of a circle and the other into the shape of a square. How should the wire be cut so that the combined area of the two figures is (a) as small as possible? (b) as large as possible?
Answer (a) $40/(\pi + 4)$ for the square and $10\pi/(\pi + 4)$ for the circle, (b) 10ft for the circle and no square.

4. Use the laws of logarithms to simplify: (a)$\ln \sqrt[3]{\frac{3e^2}{5}}$, (b) $\log_5 \frac{1}{\sqrt{45}}$
Answer: (a)$\frac{1}{3}(2 + \ln 3 - \ln 5)$, (b) $-\frac{1}{2} - \log_5 3$.

5. Compute the following derivatives: (a) $D_x(4\cos(\sin 3x))$, (b) $D_x(\sqrt[5]{e^{3x}})$, (c) $D_x(e \tan(2\pi x))$.
Answer: (a) $-12 \cos 3x \sin(\sin 3x)$, (b)$\frac{3}{5}\sqrt[5]{e^{3x}}$, (c) $\frac{2e\pi}{\cos^2(2\pi x)} = 2e\pi \sec^2(2\pi x)$.