Math 111 - Fall 2008 - Solutions for selected homework problems

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1. (Section 1.2, 74)

$$f(x) = \begin{cases} 60t & \text{for } 0 \le t \le 1 \\ 60 & \text{for } 1 < t \le 3/2 \\ 60 + 60(t - 3/2) & \text{for } 3/2 < t \le 5/2 \end{cases}$$

The $t - 3/2$ appears in the last row because 3/2 h is the new reference time after the stop.

2. (Section 1.3, 1 in Questions and Discussion)
A polynomial has general form $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$, with $a_0,...,a_n \in \mathbb{R}$. If $a_n \neq 0$ then the degree of the polynomial P is n. Positive degree means $n \geq 1$. A polynomial of 0 degree is a constant, a_0.
(a) A constant polynomial $P(x) = a_0$, with $a_0 > 0$
(b) Any up side down parabola, with vertex below the x-axis e.g. $P(x) = -x^2 - 5$. This has positive degree $2 > 0$.
(c) This is impossible. If the leading coefficient is > 0 as $x \to \infty$, $P(x) \to \infty$ and therefore the graph cannot stay below the x-axis. A constant polynomial doesn't work as it doesn't have positive degree.
(d) This is impossible. As $x \to \infty$, $P(x) \to -\infty$ and as $x \to -\infty$, $P(x) \to \infty$. As a polynomial is continuous the graph has to intersect the x-axis. Think of $P(x) = -5x^3$.
(e) $P(x) = 1/2$
(g) $1/x$
(h) $f(x) = \frac{x^2+1}{x^4+1}$. Nonconstant means it has to depend on x. The function $f(x)$ is never 0 because $x^2 + 1$ is never 0 and the graph doesn't have no vertical asymptotes because $x^4 + 1$ is never 0.

3. (Section 2.1, 30)
$A(x) = x(50 - x)$. If $(a, A(a))$ is a point where the tangent is horizontal, the slope of the tangent is 0 here, that is we need to set

$$\lim_{x \to a} \frac{A(x) - A(a)}{x - a} = 0$$

and solve for a.

$$\lim_{x \to a} \frac{A(x) - A(a)}{x - a} = \lim_{x \to a} \frac{x(50 - x) - a(50 - a)}{x - a} = \lim_{x \to a} \frac{50x - x^2 - 50a + a^2}{x - a}$$

$$= \lim_{x \to a} \frac{50(x - a) - (x^2 - a^2)}{x - a} = \lim_{x \to a} \frac{50(x - a) - (x + a)(x - a)}{x - a} = \lim_{x \to a} \frac{(x - a)(50 - (x + a))}{x - a} =$$

$$= \lim_{x \to a} (50 - x - a) = 50 - 2a$$

Setting $50 - 2a = 0$ we get $a = 25$ and the corresponding point on the graph $(25, 625)$.

4. (Section 2.2, 32). Whenever you see such a limit, with square roots, it's good to think of multiplying by the conjugate expression. The conjugate of $3 - \sqrt{x}$ is $3 + \sqrt{x}$. We get:

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{9 - x} = \lim_{x \to 9} \frac{3 - \sqrt{x}}{9 - x} \cdot \frac{3 + \sqrt{x}}{3 + \sqrt{x}} = \lim_{x \to 9} \frac{9 - x}{(9 - x)(3 + \sqrt{x})} = \lim_{x \to 9} \frac{1}{3 + \sqrt{x}} = 1/6.$$

We had to also multiply the denominator with $3 + \sqrt{x}$ so that we don't change the expression.