MATH 111-FALL 2008: PRACTICE FOR MIDTERM EXAM I + ANSWERS

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1. Find the domain and range of the function $f(x) = \tan(2x)$.

2. Draw the graphs of $f(x) = \frac{1}{x^2}$ and $g(x) = \cos x$.

3. Find the domain and range of $f(x) = \sqrt{|\sin(x)|}$

4. Let $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{x-2}$. Compute $(f \circ g)(x)$ and $(g \circ f)(x)$ for those values of x for which they make sense.

5. Decide if the following statements are true or false. If a statement is true just write TRUE next to it. If a statement is false write FALSE next to it and either explain why it is false or provide a counterexample.

A) The graph of the function $f(x) = \frac{1}{x^2-x}$ has a vertical asymptote at $x = 1$.
B) The function $f(x) = |x+1|$ is even.
C) If a function f is continuous at a then f has a limit at a.
D) If

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$$

then f is continuous at $x = 1$.

6. Give an example of a function which has a limit but is not continuous at $x = 2$.

7. Evaluate

$$\lim_{t\to 0} \frac{1}{t^2} \sin^2(\frac{t}{2})$$

$$\lim_{x\to -2^-} \frac{x+2}{|x+2|}$$

8. Using the intermediate value property show that the equation $\sqrt{x} + x = 1$ has at least one solution.

8. Find the equation of the line tangent at the point $(2, \frac{1}{2})$ to the graph of $f(x) = \frac{1}{x}$.
9. Let $f(x) = \frac{1}{x-3}$. Apply the definition of the derivative to find $f'(x)$.

10. At what values of x is the function $f(x) = x^2 - 3$ increasing?

11. A car is traveling at 100 ft/s when the driver suddenly applies the breaks ($x = 0, t = 0$). The position function of the skidding car is $x(t) = 100t - 5t^2$. How far and for how long does the car skid before it comes to a stop?

12. A snowball with an initial radius of 12cm melts and its radius decreases at a constant rate. It begins to melt at $t = 0$ and it takes 12h to disappear. What is the rate of change of its volume when $t = 6$?

Answers:

1. Domain(f)=$\{x, \cos(2x) \neq 0\} = \{x, 2x \neq \frac{\pi}{2}(2k+1), k \in \mathbb{Z}\} = \{x, x \neq \frac{\pi}{4}(2k+1), k \in \mathbb{Z}\}$, Range($f$)=$\mathbb{R}$.

3. Domain(f)=$\mathbb{R}$, Range(f)=[0,1].

5. A) TRUE, B) FALSE, C) TRUE, D) FALSE

7. 1/4, -1

8. Take $f(x) = \sqrt{x} + x - 1$ and evaluate it at $x = 4$ and $x = 0$. Then apply IVP.

8. bis. $y = -1/4x + 1$.

9. $-\frac{1}{(x-3)^2}$.

10. The function f is increasing if the derivative is positive(that is there is a positive change in the y-values). Compute $f'(x) = 2x$ and set $2x \geq 0$. This implies f is increasing for $x \geq 0$. It is strictly increasing for $x > 0$.

11. 10 sec, 500 ft

12. Volume $= \frac{4}{3}\pi r^3$, where r=radius.When $t = 6$ the radius is 6cm (because it melts at constant rate), so we need to compute $V'(6)$. Computing with the definition yields $V'(6) = 144\pi$.