Rice University
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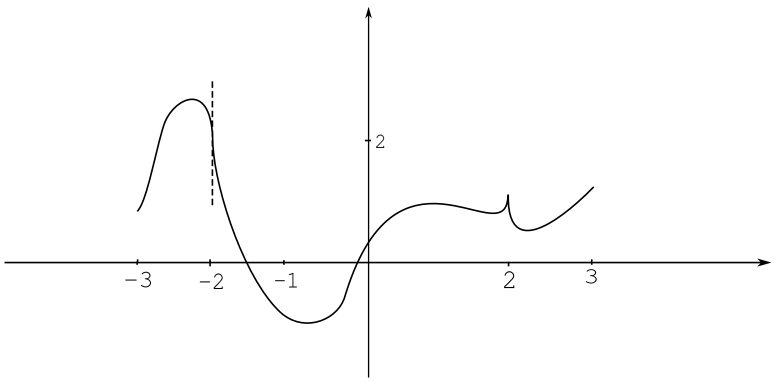
Math 111 - Fall 2008
Midterm Exam 2
SOLUTIONS

1. *(12 points)* Below you are given the graph of the function f defined on [-3,3], together with the tangent line at (-2,2). Decide if the following statements are true or false. If a statement is true, just write TRUE next to it. If a statement is false, write FALSE and briefly explain why you think it is false.



A) $f'(-1) = 1$. FALSE. The slope of the tangent line at $(-1, f(-1))$ is negative.

B) f is differentiable at $x = -2$. FALSE. The slope is undefined at $x = -2$.

C) f has a local maximum at $x = 2$. TRUE. $f(2) \geq f(x)$ for x close enough to 2.

D) f has a unique local minimum FALSE. There are three local minima: $f(3)$, $f(a)$ for a value a between -1 and 0, $f(b)$ for a value b between 2 and 3.

2.*(12 points)* Compute the following derivatives.

A) If $f(x) = \sqrt{x^2 + \frac{1}{x^2}}$ then $f'(x) = \frac{1}{2}(x^2 + x^{-2})^{-1/2} \cdot (2x - \frac{2}{x^3}) = (x^2 + x^{-2})^{-1/2} \cdot (x - \frac{1}{x^3})$

B) $D_x(\frac{\sin 4x}{x+1}) = \frac{D_x(\sin 4x)\cdot(x+1) - \sin 4x \cdot D_x(x+1)}{(x+1)^2} = \frac{4(\cos 4x)(x+1) - \sin 4x}{(x+1)^2}$ (quotient rule)

C) $D_x(\cos 5x \cdot e^{4x}) = D_x(\cos 5x) \cdot e^{4x} + \cos 5x \cdot D_x(e^{4x}) = -5 \cdot \sin 5x \cdot e^{4x} + 4 \cdot \cos 5x \cdot e^{4x}$ (product rule)

D) $g(x) = 2^{\sqrt{x}}$.What is $g'(4)$?

$g'(x) = 2^{\sqrt{x}} \cdot \ln 2 \cdot D_x(\sqrt{x}) = 2^{\sqrt{x}} \cdot \ln 2 \cdot \frac{1}{2\sqrt{x}}$.

$g'(4) = 2^{\sqrt{4}} \cdot \ln 2 \cdot \frac{1}{2\sqrt{4}} = 4 \cdot \ln 2 \cdot \frac{1}{4} = \ln 2$.

3. *(12 points)* Simplify the following expression:

A) $\frac{\log_3 25}{\log_3 125} = \frac{\log_3 5^2}{\log_3 5^3} = \frac{2 \log_3 5}{3 \log_3 5} = \frac{2}{3}$

B) $\ln(2 \cdot e^5) = \ln 2 + \ln e^5 = \ln 2 + 5$

C) $\log_2 \sqrt[3]{(\frac{1}{2})^5} = \log_2(\frac{1}{2})^{5/3} = \log_2 2^{-5/3} = -\frac{5}{3}$

4. *(12 points)* An oil field containing 20 wells has been producing 4000 barrels of oil daily (200 barrels per well). For each new well drilled, the production of each well decreases by 5 barrels. How many new wells should be drilled to maximize the total daily production of the oil field?

If t extra wells are drilled, there will be a total of $20 + t$ wells, each producing $200 - 5t$ barrels per day. There may be added as few as 0 wells and as many as 40 wells(to keep the production per well, $200 - 5t$, positive).
We need to maximize the daily production given by $P(t) = (20 + t)(200 - 5t) = 4000 - 100t + 200t - 5t^2 = 4000 + 100t - 5t^2$, for $t \in [0, 40]$.

Look for critical points of P.
$P'(t) = 100 - 10t = 0$ implies t=10.
Possible values for t: 0, 10, 40.
$P(0) = 4000$, $P(10) = 4000 + 100 \cdot 10 - 5 \cdot 10^2 = 4500$, $P(40) = 0$.
Thus the production is maximized exactly when $t = 10$ more wells are drilled.

5. *(12 points)* A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from the point where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?

Let x be the variable height of the balloon. Using the Pythagorean Theorem, the distance between the observer and the balloon is given by

$$D(x) = \sqrt{x^2 + 300^2}$$

Both x and D change with time t and we are asked to find $\frac{dD}{dt}$ at the specified moment, when $x = 400$.

$$\frac{dD}{dt} = \frac{dD}{dx} \cdot \frac{dx}{dt} = \frac{2x}{2\sqrt{x^2 + 300^2}} \cdot \frac{dx}{dt} = \frac{x}{\sqrt{x^2 + 300^2}} \cdot \frac{dx}{dt} =$$

$$= \frac{400}{\sqrt{400^2 + 300^2}} \cdot 5 = \frac{400}{\sqrt{250000}} \cdot 5 = \frac{400}{500} \cdot 5 = 4m/sec.$$