First Midterm Exam
Math 212 Fall 2010

Instructions: This is a **90 minute** exam. You should work alone, without access to any book or notes. No calculators are allowed. Do not discuss this exam with anyone other than your instructor. When you have completed the exam, write out and sign the Honor Code pledge on the front.

The exam consists of 6 questions. You must show all of your work on each problem to receive full credit, and **be sure to clearly indicate your final answer** to each question.

Name:

Write out the Honor Pledge:

Signature:

Problem	Score
1	/15
2	/15
3	/20
4	/15
5	/15
6	/20
Total	

1. [15 Points]

(a) Compute the dot product $(3\mathbf{i} - 2\mathbf{k}) \cdot (\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})$.

(b) Compute the determinant $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 0 & 0 \\ 3 & 1 & -2 \end{vmatrix}$.

(c) Let $\mathbf{u} = (1, -1, 2)$, $\mathbf{v} = (1, -3, 1)$, and $\mathbf{w} = (2, -2, 4)$. Compute $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

2. [15 Points]

(a) Let $f(x, y, z) = x \sin yz + z^3 \ln y$. Compute the gradient ∇f.

(b) Let $\mathbf{g}(x, y, z) = (xy - e^{yz}, z \sin xy^2)$. Compute the matrix of partial derivatives $\mathbf{Dg}$.

3. [20 Points] Suppose that the temperature at a point (x, y), $x > 0$ in the plane is given by $T = \sqrt{x} + e^{x^2 y}$ and that an object is moving in the plane according to the path $\mathbf{c}(t) = (1 - \cos 3t, \cos t)$. Let $T(t)$ be the temperature the object experiences at time t.

(a) Using the multivariable chain rule, find the rate of change $\frac{dT}{dt}$ of the temperature experienced by the object at time $t = \pi/2$.

(b) Find an approximate value for the temperature at time $t = (\pi/2) + 0.06$.

4. [15 Points] Compute the following multivariable limits, or show that they do not exist:

(a) $$\lim_{(x,y)\to(0,0)} \frac{3x}{x^2+y^2}$$

(b) $$\lim_{(x,y)\to(0,\pi/3)} \frac{\cos(xy^2)-1}{xy^2}$$

(c) $\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4+y^6}$

5. [15 Points]

(a) Find the equation for a plane that is perpendicular to both the plane $x+3y-2z+4=0$ and the plane $x+2z=0$, or show that no such plane exists.

(b) Find the equation for a plane that is perpendicular to both the plane defined by $2x+y-3z+1=0$ and the line $\mathbf{l}(t)=(1,-2,-7)+t(2,5,3)$, or show that no such plane exists.

6. [20 Points] In this problem, we consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = |x| + |y|.$$

(a) Plot the level sets $f(x, y) = c$ for $c = -1, 0, 1, 2$.

(b) Sketch the graph of f.

(c) Find an equation for the tangent plane to the graph of f at the point $(1, -2, 3)$.