

Third Midterm Exam
Math 212 Fall 2010

Instructions: This is a 120 minute exam. You should work alone, without access to any book or notes. **No calculators are allowed.** Do not discuss this exam with anyone other than your instructor. When you have completed the exam, write out and sign the Honor Code pledge on the front.

The exam consists of 6 questions. You must show all of your work on each problem to receive full credit, and **be sure to clearly indicate your final answer** to each question.

Name:

Write out the Honor Pledge:

Signature:

Problem	Score
1	/20
2	/15
3	/15
4	/20
5	/15
6	/15
Total	

1. [20 Points] Let $\Phi(u, v) = (u \cos v, u^2, u \sin v)$ be a parameterized surface, where $0 \leq u \leq 3$ and $0 \leq v \leq 2\pi$.

(a) Compute the tangent vectors $\mathbf{T}_u(u, v)$ and $\mathbf{T}_v(u, v)$ associated to this parametrization.

(b) Find a unit vector $\mathbf{n}(u, v)$ orthogonal to the surface at the point $\Phi(u, v)$.

(c) Find an equation for the tangent plane to the surface at the point $(\sqrt{2}, 4, \sqrt{2})$.

(d) Compute the area of the surface.

2. [15 Points] Let $C \subset \mathbb{R}^2$ be the arc of the parabola $y = x^2$ from $(1, 1)$ to $(3, 9)$.

(a) Write a parametrization $\mathbf{c}(t)$ that traces out the arc C for $0 \leq t \leq 1$.

(b) Compute the path integral $\int_C \frac{y}{x} ds$.

(c) Let $f(x, y) = \frac{e^x}{x^2 + y^2}$. Compute the line integral $\int_{\mathbf{c}} (\nabla f) \cdot d\mathbf{s}$.

3. [15 Points] Let S be the surface of the tetrahedron (triangular pyramid with four triangular faces) with vertices at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$. Compute the surface integral

$$\int_S xyz \, dS.$$

4. [20 Points] Let $D^* \subset \mathbb{R}^2$ be rectangle $-1 \leq u \leq 1$, $0 \leq v \leq 1$, and let

$$\mathbf{T}(u, v) = (x(u, v), y(u, v)) = (uv, u^2).$$

(a) Sketch the image $D = \mathbf{T}(D^*)$ in the xy plane.

(b) Is the transformation \mathbf{T} one-to-one on D^* ?

- (c) Compute the double integral $\iint_D x^2 \, dx \, dy$ using the change of variables formula for double integrals.

5. [15 Points] Let $\mathbf{F}(x, y, z) = (3x^2 + 2xy)\mathbf{i} + (2yz^2 + x^2 + 3y)\mathbf{j} + (2y^2z)\mathbf{k}$.

(a) Compute $\nabla \cdot \mathbf{F}$.

(b) Compute $\nabla \times \mathbf{F}$.

(c) Let $\mathbf{c}(t) = (\cos t, \sin t, 0)$, for $0 \leq t \leq \pi$. Compute the line integral

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}.$$

6. [15 Points] A giant foam ball of radius 5 m has non-uniform density $\delta(d) = \frac{1}{d}$ kg/m³, where d is the distance in meters from the center of the ball. Suppose a cylindrical hole of radius 3 m has been cut straight through the center of the ball (i.e. the axis of the cylinder passes through the center of the ball). What is the mass of the remaining part of the ball?

