MATH 366: Assignment 1

Due Friday, January 15, 2010

If you don't own an actual compass and straightedge (or even if you do), you may want to look into using a computer program to help experiment with compass and straightedge constructions; you may have already used one of these programs in high school. See Wikipedia's list of interactive geometry software. (Personally, I've been using C.a.R. (Compass and Ruler), a free open source dynamic geometry program.)

1. Describe how to construct each of the following with a compass and straightedge. (You need not prove that your constructions are correct, only give a precise description in words of your construction. It would be helpful for you to provide pictures of your construction as well, but your written description should stand alone in the sense that someone ought to be able to follow your instructions without any figure.)

   (a) Given a segment AB. Construct an equilateral triangle with AB as a side.

   (b) Given a segment AB. Construct the perpendicular bisector of AB.

   (c) Given a line l and a point P lying on l. Construct the line through P perpendicular to l.

   (d) Given a line l and a point P *not* lying on l. Construct the line through P perpendicular to l.

   (e) Given a line l and a point P not lying on l. Construct a line through P parallel to l.

   (f) Given an angle. Construct the bisecting ray of the angle.

   (g) Given $\triangle ABC$ and a segment $DE \cong AB$. Construct a point F on a given side of the line $\overleftrightarrow{DE}$ such that $\triangle DEF \cong \triangle ABC$.

   (h) Given angle $\angle ABC$ and ray $\overrightarrow{DE}$. Construct F on a given side of line $\overleftrightarrow{DE}$ such that $\angle ABC \cong \angle FDE$.

   [For hints, see page 46-47 in the textbook.]

2. Euclid assumed the the compass to be *collapsing*. That is, given two points P and Q, the compass can draw a circle with center P passing through Q; however, the spike cannot me moved to another center O to draw a circle of the same radius. Once the spike is moved, the compass collapses.

   (a) Check through your solutions in the previous exercise to see whether they are possible with a collapsing compass.

   (b) Given three points P, Q, and R. Construct with a straightedge and collapsing compass a rectangle $\square PQST$ with PQ as a side and such that $PT \cong PR$.

   (c) Given a segment PQ and a ray $\overrightarrow{AB}$. Construct with a straightedge and collapsing compass the point C on $\overrightarrow{AB}$ such that $PQ \cong AC$.

This shows that you can transfer segments with a collapsing compass and straightedge, and hence that anything which can be constructed with a straightedge and lockable compass can still be constructed with a straightedge and collapsing compass.

3. What is the flaw in the "proof" that all triangles are isosceles in pages 25-27 in the textbook? (All the theorems of Euclidean geometry quoted by the argument are correct.)

4. The number $\phi = (1 + \sqrt{5})/2$ is called the *golden ratio*, and a rectangle whose sides are in this ratio is called a *golden rectangle*. Show that a golden rectangle can be constructed with a compass and straightedge as follows:

   (a) Construct a square $\square ABCD$.

   (b) Construct the midpoint M of AB.

   (c) Construct the point E on the line $\overleftrightarrow{AB}$ such that B is between A and E, and $MC \cong ME$.

   (d) Construct the foot F of the perpendicular from E to $\overleftrightarrow{DC}$.

   (e) Then $\square AEFD$ is a golden rectangle. [Hint: use the Pythagorean theorem for $\triangle MBC$.]

   (f) Moreover, $\square BEFC$ is another golden rectangle. [Hint: show that $1/\phi = \phi - 1$.]