

# MATH 366: Assignment 2

Due Friday, January 22, 2010

## Writing definitions

1. Recall that our axioms for geometry will eventually involve five undefined terms: *point*, *line*, *incident*<sup>1</sup>, *between*<sup>2</sup>, and *congruence*<sup>3</sup>.

Using these undefined terms, we can define various other terms and notation. For example, in class we said that three points  $A$ ,  $B$ , and  $C$  are *collinear* if there exists a line  $l$  so that  $A \perp l$ ,  $B \perp l$ , and  $C \perp l$ . Likewise we could define  $\overleftrightarrow{AB}$  to be a line which passes through  $A$  and  $B$ .<sup>4</sup>

Using the undefined terms, write precise mathematical definitions for the following:

- (a) Given distinct points  $A$  and  $B$ , define the *line segment*  $AB$ .
- (b) Given distinct points  $A$  and  $B$ , define the *ray*  $\overrightarrow{AB}$ .
- (c) Given non-collinear distinct points  $A$ ,  $B$ , and  $C$ , define the *angle*  $\angle ABC$ .
- (d) Define a *right angle*.
- (e) Given a line  $l$  and points  $A$  and  $B$  not lying on  $l$ , define what it means for  $A$  and  $B$  to be *on the same side* of  $l$ .
- (f) Define what it means for a point  $P$  to be in the *interior* of an angle  $\angle ABC$ .
- (g) Define the *midpoint* of a segment  $AB$ .
- (h) Define the *perpendicular bisector* of a segment  $AB$ .
- (i) Define what it means for a ray  $\overrightarrow{BD}$  to *bisect* the angle  $\angle ABC$ .
- (j) Given two distinct points  $A$  and  $B$ , define the *circle* with center  $A$  and radius  $AB$ .
- (k) Given three distinct points  $A$ ,  $B$ , and  $C$ , define the *triangle*  $\triangle ABC$ .
- (l) Define the *vertices*, *sides*, and *angles* of a triangle  $\triangle ABC$ .
- (m) Define the *interior* of  $\triangle ABC$ .
- (n) Define the *medians* of a triangle.
- (o) Define the *altitudes* of a triangle.
- (p) Define *isosceles triangle*, its *base*, and its *base angles*.

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<sup>1</sup>We express incidence by saying that a point  $P$  lies on a line  $l$  or equivalently that a line  $l$  passes through a point  $P$ , or equivalently we write  $P \perp l$ .

<sup>2</sup>We might say a point  $A$  is *between* points  $B$  and  $C$  or write  $B * A * C$ . You should think of betweenness as only making sense for three collinear points.

<sup>3</sup>Congruence will really be two separate undefined relations: congruence between two segments and congruence between two angles.

<sup>4</sup>Of course, for this not to be terrible notation, we would need an axiom or theorem stating that such a line exists and is unique (this is our incidence axiom I1). For this problem, try to keep track of what statements need to be true for your definitions to make sense.

- (q) Define *equilateral triangle*.
- (r) Define *right triangle*.
- (s) Define *quadrilateral*.
- (t) Define *rectangle*.

[If you're unfamiliar with what any of these terms mean, ask a friend or check the book: I've taken this exercise largely from pages 42-45 of the book, which includes pictures of some of these terms.]

### Logic

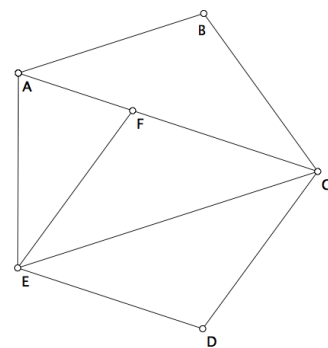
- 2. Do exercises 1, 2, 3, 17, and 19 in chapter 2 of the textbook.

### Incidence geometry

- 3. Give careful rigorous proofs of Propositions 2.3, 2.4, and 2.5 in the textbook.
- 4. (a) Show that up to isomorphism<sup>5</sup>, there is only one incidence geometry with exactly three points (i.e. show that any incidence geometry with exactly three points is isomorphic to our example from class).
- (b) Classify up to isomorphism the incidence geometries with exactly four points (i.e. give some examples of incidence geometries with exactly four points, show that they aren't isomorphic to each other, and show that every incidence geometry with exactly four points is isomorphic to one of your examples).

### Compass and straightedge construction

- 5. Describe how to construct a regular hexagon and a regular octagon using a compass and straightedge.<sup>6</sup>
- 6. For this problem, you may use anything you know about about Euclidean geometry, including the fact that the sum of the angles of a triangle is  $180^\circ$ , facts about similar triangles, etc. Let  $ABCDE$  be a regular pentagon.
  - (a) Find the degree measures of the angles of  $\triangle ACE$  and the regular pentagon  $ABCDE$ .
  - (b) Let  $F$  be the point where the bisector of  $\angle AEC$  meets the segment  $AC$ . Show that  $\triangle AEF$  is similar to  $\triangle ACE$  and that  $\triangle EFC$  is isosceles.
  - (c) Show that the ratio between the lengths of segments  $AC$  and  $AE$  is the golden ratio  $\phi = (1 + \sqrt{5})/2$ . (An isosceles triangle such as  $\triangle ACE$  with this property is called a *golden triangle*.)
  - (d) Given a segment  $AB$ , describe how to construct using a compass and straightedge a regular pentagon with sides congruent to  $AB$ .



<sup>5</sup>For the definition and some examples of isomorphism, see pages 79-81 in the textbook.

<sup>6</sup>For these problems, you need not describe every circle and line to be drawn: you're allowed to use the compass and straightedge constructions from the Assignment 1 in your construction.