

# MATH 499: Homework 6

October 8, 2009

You may wish to use a computer to do most of the work in the following calculations.<sup>1</sup>

1. (a) Determine whether or not  $f = xy^3 - z^2 + y^5 - z^3$  is in the ideal

$$I = \langle -x^3 + y, x^2y - z \rangle.$$

- (b) Determine whether or not  $f = x^3z - 2y^2$  is in the ideal

$$I = \langle xz - y, xy + 2z^2, y - z \rangle.$$

2. (a) Find the points on the variety

$$V(x^2 + y^2 + z^2 - 1, x^2 + y^2 + z^2 - 2x, 2x - 3y - z).$$

- (b) Find the points on the variety

$$V(x^2y - z^3, 2xy - 4z - 1, z - y^2, x^3 - 4zy).$$

3. (a) Find an implicit equation for the surface parametrized by:

$$\begin{aligned}x &= ut \\y &= 1 - u \\z &= u + t - ut\end{aligned}$$

- (b) Find an implicit equation for the surface parametrized by:

$$\begin{aligned}x &= t + u \\y &= t^2 + 2tu \\z &= t^3 + 3t^2u\end{aligned}$$

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<sup>1</sup>There are other programs, such as Singular and Macaulay 2 that are available for free and may be able to do some things with Gröbner bases that Mathematica can't. For example, I don't know how to get Mathematica to express the generators of the Gröbner basis as linear combinations of the original generators, which I can do in Macaulay 2 with the `getChangeMatrix` command.