1. Compute the resultant of \( f(x) = x^5 - 3x^4 - 2x^3 + 3x^2 + 7x + 6 \) and \( g(x) = x^4 + x^2 + 1 \). Do these polynomials have a common factor in \( \mathbb{Q}[x] \)?

2. Let \( f, g \in \mathbb{C}[x] \) be polynomials of degree 3. Verify in this case that the following are equivalent:
   
   (a) The polynomials \( f \) and \( g \) have a non-constant common factor.
   
   (b) \( \text{Res}(f, g, x) = 0 \).

3. Resultants give another method of finding an implicit polynomial equation for a parametric rational curve, in the following way: given a rational curve defined parametrically by
   
   \[
   (x, y) = \left( \frac{a(t)}{p(t)}, \frac{b(t)}{q(t)} \right),
   \]

   we can find an implicit equation by calculating the resultant \( \text{Res}(f, g, t) \) of the polynomials \( f = a(t) - xp(t) \) and \( g = b(t) - yq(t) \), where we are regarding \( f \) and \( g \) as polynomials in \( t \) with coefficients in \( \mathbb{C}[x, y] \).
   
   (a) Show that \( \text{Res}(f, g, t) \), as a polynomial in \( x \) and \( y \), vanishes on the parametric curve.
   
   (b) Use this method to find an implicit equation for the following curves:
   
   i. \( x = t^2 \), \( y = t^2(t + 1) \).
   
   ii. \( x = \frac{t-1}{t^2} \), \( y = t - 1 \).
   
   iii. \( x = \frac{t(t^2+1)}{t^4+1} \), \( y = \frac{t(t^2-1)}{t^4+1} \).
   
   (c) Use the Gröbner basis method to find implicit equations for the above parametric curves and check that they define the same curves.

4. Consider the following polynomials in \( k[x, y] \):
   
   \[
   f = x^2y - 3xy^2 + x^2 - 3xy \\
   g = x^3y + x^3 - 4y^2 - 3y + 1
   \]

   (a) Compute \( \text{Res}(f, g, x) \).
   
   (b) Compute \( \text{Res}(f, g, y) \).
   
   (c) What do your answers from (a) and (b) tell you about \( f \) and \( g \)?