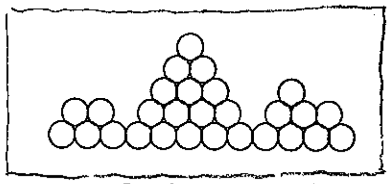
MATH 468 Handout #11 March 19, 2014

HW 7: due Wednesday March 26

Define an ***n-fountain*** to be an arrangement of equal disks in rows such that the bottom row has n disks with no gaps, and such that each disk in a higher row touches exactly two disks in the next lower row. Here's a picture of a 12-fountain:



1. Determine, with proof, the number of n-fountains for every positive integer n.

2. Determine, with proof, the number of n-fountains which are *gapless*. This means that each row consists of just one string of disks without gaps.

3. From the Bonus problems in Chapter 9:

59 Let $\Theta_n(t) = \sum_k e^{-(k+t)^2/n}$, a periodic function of t. Show that the expansion of $\Theta_n(t)$ as a Fourier series is

$$\Theta_n(t) = \sqrt{\pi n}\big(1 + 2e^{-\pi^2 n}(\cos 2\pi t) + 2e^{-4\pi^2 n}(\cos 4\pi t) + 2e^{-9\pi^2 n}(\cos 6\pi t) + \cdots\big).$$

(This formula gives a rapidly convergent series for the sum $\Theta_n = \Theta_n(0)$ in equation (9.93).)

You may assume as known the formula for the Fourier transform of the Gaussian function:

$$\int_{-\infty}^{\infty} e^{-t^2} e^{-2\pi i a t}\,dt = \sqrt{\pi}\, e^{-\pi^2 a^2}.$$

4. Also complete Exercise B from the syllabus. (This was actually due on Monday, but I forgot to remind you.)