MATH 468 Handout #12 March 26, 2014

HW 8: due Wednesday April 2

1. On page 121 there appears the equation $f(RS) = f(S) + 1$. What is the corresponding equation for $f(LS)$?

2. Suppose that $\frac{p}{q}$ is a fraction in the Stern–Brocot tree. Prove that $\frac{p}{p+q}$ and $\frac{p+q}{q}$ are in the next row in the Stern–Brocot tree.

3. If $\frac{p}{q}$ is a fraction in the Stern–Brocot tree and $p \perp q$, define the <u>simplicity</u> of this fraction to be $\frac{1}{pq}$. Find a formula for the sum of the simplicities in any row of the Stern–Brocot tree.

(Example row 3: $\frac{1}{4}, \frac{2}{5}, \ldots, \frac{4}{1}$ yields the sum
$\frac{1}{4} + \frac{1}{10} + \frac{1}{15} + \frac{1}{12} + \frac{1}{12} + \frac{1}{15} + \frac{1}{10} + \frac{1}{4} = 1$.)

4. Consider two <u>adjacent</u> fractions $\frac{p}{q}$ and $\frac{p'}{q'}$ in the mth row of the Stern–Brocot tree. Their <u>mediant</u> is of course $\frac{p+p'}{q+q'}$. Prove that $p+p'$ and $q+q'$ are <u>not</u> relatively prime. And prove that $\frac{p+p'}{q+q'}$ is the common ancestor of $\frac{p}{q}$ and $\frac{p'}{q'}$ in the row closest to m.

Example: $\frac{7}{11}$ and $\frac{7}{10}$ in the 5th row, with ancestor $\frac{14}{21} = \frac{2}{3}$ in the 2nd row.