MATH 468 Handout #2 January 22, 2014

The tables 155 and 258 and 259 all have the following features:

- The entries are indexed by a row number n and a column number k
- Both n and k are nonnegative integers
- For $n < k$, the entries are all 0
- For $n = k$, the entries are all 1

That is, these tables represent matrices of infinite order which are *lower triangular.*

HW 2: due Wednesday January 29.

1. Show that matrices with the above properties form a *group* with respect to the usual definition of matrix multiplication.

Let's temporarily call this group G .

> CONTEST: Either find out that this group has a well known name, or make one up. This contest comes with a PRIZE for the best suggestion.

For a matrix A in G, form a new matrix by multiplying the entry of A in row n and column k by $(-1)^{n-k}$. Denote this new matrix by the symbol $A_\pm$.

NOTATIONS: **B** = binomial coefficient matrix of Table 155
S = "subset" matrix of Table 258
C = "cycle" matrix of Table 259

2. Prove that $S^{-1} = C_\pm$ and $C^{-1} = S_\pm$.

3. What is B^{-1}? **(Prove it!)**

4. Prove the formula (5.30) on page 171. If you wish, you may use equation (5.55).

5. Prove formula (6.24) in Table 265. (I like this result as it combines all three types of the numbers that we have studied.)