MATH 468 Handout #4 February 5, 2014

EXAM 1: due Wednesday February 12.

1. Prove that the product of any n consecutive integers is divisible by $n!$

2. Solve the book's exercise 1-10:

10 Let Q_n be the minimum number of moves needed to transfer a tower of n disks from A to B if all moves must be *clockwise* — that is, from A to B, or from B to the other peg, or from the other peg to A. Also let R_n be the minimum number of moves needed to go from B back to A under this restriction. Prove that

$$Q_n = \begin{cases} 0, & \text{if } n = 0; \\ 2R_{n-1}+1, & \text{if } n > 0; \end{cases} \qquad R_n = \begin{cases} 0, & \text{if } n = 0; \\ Q_n+Q_{n-1}+1, & \text{if } n > 0. \end{cases}$$

(You need not solve these recurrences; we'll see how to do that in Chapter 7.)

3. Prove the relation

$$F_n^2 - F_{n+k}F_{n-k} = (-1)^{n-k}F_k^2.$$

(Notice that the case $k = 1$ is Cassini's identity.)

This exam is under the same rules as the homework assignments, except that you must work alone. **Be sure to write and sign the honor pledge.**