MATH 468 Handout #5 February 12, 2014

HW 4: due Wednesday February 19.

- From Chapter 1, Homework exercise 10:

10 Let Q_n be the minimum number of moves needed to transfer a tower of n disks from A to B if all moves must be *clockwise* — that is, from A to B, or from B to the other peg, or from the other peg to A. Also let R_n be the minimum number of moves needed to go from B back to A under this restriction. Prove that

$$Q_n = \begin{cases} 0, & \text{if } n = 0; \\ 2R_{n-1}+1, & \text{if } n > 0; \end{cases} \quad R_n = \begin{cases} 0, & \text{if } n = 0; \\ Q_n+Q_{n-1}+1, & \text{if } n > 0. \end{cases}$$

(You need not solve these recurrences; we'll see how to do that in Chapter 7.)

Now solve for all the Q_n .

- Chapter 6, Homework exercise 42

42 If S is a set of integers, let $S + 1$ be the "shifted" set $\{x + 1 \mid x \in S\}$. How many subsets of $\{1, 2, \ldots, n\}$ have the property that $S \cup (S + 1) = \{1, 2, \ldots, n + 1\}$?

- Chapter 7, Homework exercise 23

23 In how many ways can a $2 \times 2 \times n$ pillar be built out of $2 \times 1 \times 1$ bricks?