MATH 468 Handout #6 February 19, 2014

HW 5: due Wednesday February 26

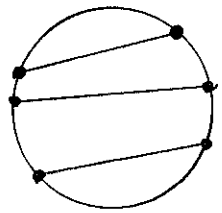
A. Define $x_0 = 1$ and for $n \geq 0$ let $x_{n+1} = 3x_n + \lfloor x_n\sqrt{5} \rfloor$. Find an explicit formula for x_n for all n.
(Putnam Competition 2007)

~~This exercise is due Monday February 24.~~

1. For $n \geq 1$ let X_n be the number of subsets T of $\{1, 2, \ldots, n\}$ which have the property that $T \cup (T+1) \cup (T+2) = \{1, 2, \ldots, n+2\}$.

Find a recursion formula for the sequence X_n and also calculate the generating function for this sequence. (Do not attempt to solve for the X_n's.)

2. In how many ways can 2n points on a circle be joined by n nonintersecting chords?



3. Discover and prove a simple formula for the Bernoulli polynomial quantity

$$B_k(x+1) - B_k(x).$$

4. Prove this formula for the Bernoulli polynomials:

$$B_k(nx) = n^{k-1} \sum_{j=0}^{n-1} B_k\left(x + \frac{j}{n}\right).$$