MATH 468      Handout #7      February 20, 2014

This morning I had some free time, so I decided to play with the following question:

**1.** How many ways are there to connect some pairs of $n$ points on a circle with nonintersecting chords? ("Some" may include "none.") Let $M_n$ denote the number of ways.

I started by drawing pictures, and found that $M_0 = 1$, $M_1 = 1$, $M_2 = 2$, $M_3 = 4$, $M_4 = 9$, $M_5 = 21$, $M_6 = 51$. And I stopped drawing.

**2.** I then went to the famous website https://oeis.org/, *The On-Line Encyclopedia of Integer Sequences*, founded in 1964 by N. J. A. Sloane. There I entered my sequence 1,1,2,4,9,21,51, and was instantly led to this entry:

A001006 Motzkin numbers: number of ways of drawing any number of nonintersecting chords joining n (labeled) points on a circle.

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1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511,
41835, 113634, 310572,853467, 2356779, 6536382, 18199284,
50852019, 142547559, 400763223, 1129760415, 3192727797,
9043402501, 25669818476, 73007772802, 208023278209, 593742784829
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(list; graph; refs; listen; history; text; internal format)

**3.** Below the numbers is a long sequence of comments, and here is one:

COMMENTS  Also number of Motzkin n-paths: paths from (0,0) to (n,0) in an n X n grid using only steps U = (1,1), F = (1,0) and D = (1,-1).

**4.** I looked up the original paper of Theodore Motzkin: "Relations Between Hypersurface Cross Ratios, and a Combinatorial Formula for Partitions of a Polygon, for Permanent Preponderance, and for Nonassociative Products." *Bull. Amer. Math. Soc.* **54**, 352-360, 1948.

That paper was dated 1948 and written from the University of Jerusalem.