Math 212 Spring 2005 **Exam #1**

Instructions: You have **two hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.
This exam consists of 6 questions. You must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. The exam is due by **Friday, 5 p.m.** Good luck!

1. Let $\mathbf{l}_1$ and $\mathbf{l}_2$ be the intersecting lines given by the parameterizations

$$\mathbf{l}_1(t) = (1,0,1) + t(0,2,1),$$
$$\mathbf{l}_2(s) = (3,3,4) + s(-2,1,-1).$$

   (a) Find the angle between $\mathbf{l}_1$ and $\mathbf{l}_2$.

   (b) Find a vector perpendicular to both $\mathbf{l}_1$ and $\mathbf{l}_2$.

2. Let $f(x,y) = 4x^2 + y^2$.

   (a) Sketch the level curves of $f(x,y) = k$ for $k = 2$, 4, 8.

   (b) On your graph from part (a), draw a vector at the point $(1,2)$ which gives the direction of the gradient of f at that point.

   (c) Graph and describe the $x = 0$, $y = 0$, and $z = 0$ cross sections of the graph of $f(x,y)$. Each cross section should appear on a separate graph.

   (d) Sketch the graph of f.

3. Suppose a particle is travelling in $\mathbb{R}^2$ such that at time t its position is given by

$$\mathbf{c}(t) = \big(t^2 - \pi t, \sin(t)\big).$$

   (a) Find the velocity of the particle at time t. At what time(s) t does the particle come to a stop?

   (b) Give a parametric equation for the tangent line to $\mathbf{c}(t)$ at time $t = \frac{\pi}{4}$.

4. The temperature at a point (x, y) on a flat metal plate is given by the function

$$T(x, y) = \frac{60}{1 + 2x^2 + 2y^2},$$

where T is measured in °C and x, y are measured in meters. Find the rate of change of temperature with respect to distance at the point $(2, 1)$ in

(a) the y-direction,

(b) the direction given by the vector $\mathbf{i} - 2\mathbf{j} = (1, -2)$,

(c) the direction of the maximum rate of change.

(d) Is there a direction at $(2, 1)$ along which T does not change? If so, find a vector pointing in that direction.

5. Let $f(x, y, z) = x^3 y^2 z$, and let $\mathbf{c}(t) = \big(e^t, \sin(t), g(t)\big)$, where $g(t)$ is a differentiable function. Use a Chain Rule from vector calculus to find

$$\frac{d}{dt} f\big(\mathbf{c}(t)\big).$$

Your final answer will be in terms of $g(t)$, $g'(t)$ and t.

6. Let

$$f(x, y) = x^3 + 3x^2 y^2 + y^3.$$

(a) Suppose a particle is sitting on the graph of f at the point $(-1, 1)$. In which direction should the particle move in order for its height to decrease most rapidly?

(b) Give the equation for the tangent plane of $f(x, y)$ at the point $(-1, 1)$.

(c) Give the quadratic Taylor polynomial for $f(x, y)$ at the point $(-1, 1)$.