

Homework 13, due Friday 4/22

1. Let $\mathbf{F}(x, y, z) = (x, y, z)$. Let S be the sphere of radius 2. Compute $\int \int_S \mathbf{F} \cdot d\mathbf{S}$.
2. Let $\mathbf{F}(x, y, z) = (x^2, y - z, 1)$. Let S be the graph of $f(x, y) = x^3 - 2xy$ with domain $D = [0, 1] \times [0, 3]$. Compute $\int \int_S \mathbf{F} \cdot d\mathbf{S}$.
3. Let C be the coke can with radius 3 and with top and bottom at $z = 1$ and $z = 0$. Give it the usual orientation. Let $\mathbf{F}(x, y, z) = (0, 0, z)$. Compute $\int \int_C \mathbf{F} d\mathbf{S}$. If you think about what is happening geometrically, then you should be able to solve this problem with barely any computations. Explain carefully what you are doing. (Hint: sketch C and \mathbf{F}).
4. p. 528, problem 3 (a), (b)
5. p. 528, problem (1), (5)
6. Compute $\int_C \mathbf{F}(x, y) \cdot d\mathbf{s}$ for $\mathbf{F}(x, y) = (x - y, y - x)$ and C the square bounded by $x = 0, x = 1, y = 0, y = 1$.
7. Compute $\int_C \mathbf{F}(x, y) \cdot d\mathbf{s}$ for $\mathbf{F}(x, y) = (\tan^{-1}(\frac{y}{x}), \ln(x^2 + y^2))$ and C the boundary of the region defined by the polar coordinate inequalities $1 \leq r \leq 2, 0 \leq \theta \leq \pi$.
8. Compute the area of the ellipse given by $\mathbf{c}(t) = (a \cos(t), b \sin(t)), t \in [0, 2\pi]$.
9. Let $\mathbf{F}(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$. Let D be the disk of radius one. Compute $\int_{\partial(D)} \mathbf{F} \cdot d\mathbf{s}$ and $\int \int_D \frac{\partial}{\partial x}(\frac{x}{x^2+y^2}) - \frac{\partial}{\partial y}(\frac{-y}{x^2+y^2}) dA$. Compare the results. Why is this NOT a counter example to Green's theorem? (read Green's theorem carefully).