Math 355 **Exam #1** **October 1, 2004**

This exam consists of 2 sections. Problems in **Part I** are True/False. You do not need to show work for these problems. Solutions to problems in **Part II**, however, must show work to earn full credit. Pledge your exam before turning it in. Good luck!

Part I: For each problem, decide whether the claim is true or false. No justification is necessary.

1. Let A, B, and C be nonsingular $p \times p$ matrices. Then it follows that
$$(ABC)^{-1} = A^{-1}B^{-1}C^{-1}.$$

2. Let A and B be $p \times p$ matrices. Then it follows that $\det(A+B) = \det(A) + \det(B)$.

3. Let A be a $p \times p$ matrix, such that the homogenous system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions. Then it follows that for any $p \times 1$ column vector $\mathbf{b}$, the system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

4. Let A be a $p \times q$ matrix, and let B and C be $q \times r$ matrices. If $AB = AC$, then $B = C$.

5. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors that solve the system $A\mathbf{x} = \mathbf{b}$. Then $\mathbf{v} + \mathbf{w}$ also solves $A\mathbf{x} = \mathbf{b}$.

6. Let T be a matrix of the form
$$T = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}.$$
Then it follows that $\det T = a_{11}a_{22}a_{33}a_{44}$.

7. Suppose A is a square matrix, and B is the matrix obtained from A by multiplying the third row of A by c. Then $\det(B) = c\det(A)$.

Part II on reverse

Part II:

1. Use Gauss-Jordan elimination to find **all** solutions to the system:

$$\begin{cases} 2x_1 + 4x_2 - 4x_3 = & 0. \\ 4x_1 + 3x_2 - 3x_3 = & -1, \end{cases}$$

2. Compute the determinant of each of the following matrices.

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 2 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 3 & -4 & 0 & 1 \\ 2 & 5 & 0 & -2 \\ -3 & 1 & 0 & -6 \\ 1 & 4 & 0 & 3 \end{bmatrix},$$

3. Let V be the vector space $\mathbb{R}^3$ with the usual operations. Which of the following are subspaces of V? Justify your answer.

   (a) The subset of all vectors $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ such that $v_1 - v_3 = 0$.

   (b) The subset of all vectors $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ such that $v_1 v_2 = 0$.

4. Consider the following augmented matrix, representing a linear system $A\mathbf{x} = \mathbf{b}$, in 5 unknowns:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 3 & 1 \\ 0 & 1 & 1 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{array}\right].$$

   (a) Explain whether this matrix is in row echelon form. If it is not, perform Gauss-Jordan elimination to put it in row echelon form.

   (b) Give the solutions to this system.

   (c) What is the rank of A? Explain your answer.

5. Let A be a square nonsingular matrix. Prove that $(A^T)^{-1} = (A^{-1})^T$.