1. Let $V$ be the vector space of all differentiable functions on $\mathbb{R}$. Consider $\varphi : V \to V$ given by $\varphi(f(x)) = e^x \cdot f(x)$. This means that to the function $f(x)$ we associate the function $e^x f(x)$.

(a) Show that $\varphi$ is a linear transformation.

(b) Determine the kernel of $\varphi$.

(c) Determine whether $\varphi$ is invertible. If $\varphi$ is invertible, what is the inverse.

2. Let $V$ be a vector space with ordered basis $B := \{v_1; v_2\}$ and $W$ a vector space with ordered basis $\tilde{B} := \{w_1, w_2, w_3\}$. Consider the linear transformation $\varphi$ with $\varphi(v_1) = w_1 - w_2 + w_3$ and $\varphi(v_2) = -w_2 + 2w_3$.

(a) What is $c_B(v_1), c_B(v_2)$?

(b) What is $c_{\tilde{B}}(w_1), c_{\tilde{B}}(w_2), c_{\tilde{B}}(w_3)$?

(c) What is $c_{\tilde{B}}(\varphi(v_1)), c_{\tilde{B}}(\varphi(v_2))$?

(d) What is the matrix representing $\varphi$ with respect to the ordered bases $B$ and $\tilde{B}$.

3. p. 261, problems 1, 2, 3.

4. Let $A$ be a $p \times q$-matrix. Consider the linear transformation $\varphi : \mathbb{R}^q \to \mathbb{R}^p$ given by $\varphi(v) = Av$. Take $\{e_1; \ldots; e_p\}$ and $\{e_1; \ldots; e_q\}$ as the ordered bases. What is the matrix representing $\varphi$ with respect to these bases.