Homework 11, due Friday 11/12

1. Let $A = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find the eigenvectors corresponding to the eigenvalues.
   (c) Find a matrix B such that $B^{-1}AB$ is a diagonal matrix.
   (d) Find A^{100}.

2. Let $A = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find the eigenvectors corresponding to the eigenvalues.
   (c) Find a matrix B such that $B^{-1}AB$ is a diagonal matrix.

3. Let $h(k)$ denote the number of hares in the year k, and $r(k)$ the number of rabbits in the year k. Assume that $h(0) = 2$ and $r(0) = 100$. Assume that the number of foxes and rabbits changes over the years according to the following rule:

$$\begin{array}{rcrcr} r(k+1) & = & 10r(k) & + & -h(k) \\ h(k+1) & = & -r(k) & + & 2h(k) \end{array}.$$

   What is the number of hares and rabbits after 2 years? After 20 years? Hint, write $\begin{pmatrix} r(k+1) \\ h(k+1) \end{pmatrix} = A \begin{pmatrix} r(k) \\ h(k) \end{pmatrix}$ for some matrix A.

4. p. 291, problem 1.

5. Show that if v is an eigenvector of the matrix A to the eigenvalue λ, then v is an eigenvector of the matrix $A^2 = A \cdot A$ to the eigenvalue λ^2.

6. Show that the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ can not be diagonalized, i.e. show that there exists no invertible matrix $P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $P^{-1}AP = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ for some numbers λ_1 and λ_2.

7. Let P be an invertible matrix. Show that A and $P^{-1}AP$ have the same characteristic polynomial.