Homework 13, due Wednesday 11/24

1. Find a Jordan matrix J and a transition matrix Q such that $J = Q^{-1}AQ$ for the matrix
$$A = \begin{pmatrix} 5 & -9 \\ 4 & -7 \end{pmatrix}.$$

2. The following matrix A has $\lambda = 1$ as an eigenvalue.
   (a) Find the other eigenvalues of A.
   (b) Find Q and J as in the previous problem:
$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ -1 & 2 & 0 \end{pmatrix}.$$

3. Find Q and J as above for the matrix $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & -2 & 0 \\ -2 & -3 & -2 \end{pmatrix}$.

4. Find Q and J as above for the matrix $A = \begin{pmatrix} 0 & 2 & 0 \\ 0 & -2 & 0 \\ -2 & -3 & -2 \end{pmatrix}$.

5. Find Q and J as above for the matrix $A = \begin{pmatrix} 4+i & 9 \\ -1 & -2+i \end{pmatrix}$.

6. Find Q and J as above for the matrix
$$A = \begin{pmatrix} 2 & 0 & -1 & -2 & 0 \\ 0 & 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

   Hint: A is upper triangular, so you know the eigenvalues right away. If you solve the problem correctly, J should have two Jordan blocks of size 1 and one Jordan block of size 3.

7. Suppose A is a 7×7 matrix with three distinct eigenvalues: $\lambda = 1$ has algebraic multiplicity 3 and geometric multiplicity 2, $\lambda = 0$ has algebraic multiplicity 2 and geometric multiplicity 1, and $\lambda = -2$ has algebraic multiplicity 2 and geometric multiplicity 2. Give a Jordan form matrix J for A. You **do not** have to find a matrix Q.