MATH 355 Section 001: Linear Algebra **Fall 2004**

Homework 15, this homework will not be graded

1. Let A be a hermitian matrix and let P be a unitary matrix. Show that $P^{-1}AP$ is hermitian.

2. Let $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$.

   (a) Show that A is similar to a diagonal matrix with real eigenvalues, i.e. find an orthogonal matrix P such that $P^{-1}AP$ is diagonal.

   (b) Which theorem tells us whether a matrix is orthogonally similar to a diagonal matrix?

   (c) Show that A is not orthogonally similar to a diagonal matrix.

3. Let $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$. The eigenvalues are $-2, 2 \pm \sqrt{2}$.

   (a) Find the Jordan form of A, i.e. find a matrix Q such that $Q^{-1}AQ$ is in Jordan form.

   (b) Apply the Gram–Schmidt algorithm to the columns of Q.

   (c) Put the new vectors from the Gram–Schmidt algorithm into a matrix, call it P. Compute $P^{-1}AP$.

   (d) Find the Schur form of A.

4. Repeat the above process for the following matrices:

   (a) $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$.

   (b) $A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$.

5. Determine whether $A = \begin{pmatrix} 1 & 2 \\ 0 & -i \end{pmatrix}$ is normal.

6. p. 330, problems 8,10.

Review problems

The exam will cover all the material taught this semester, with a stress on the material covered since the last exam. For problems on the old material look at the old homework and the old review sheet.
For the new material, do the above homework carefully, it covers most of the material.

1. p. 303, problems 10,11, 13 (ignore the last part).
2. Find an orthonormal basis for the subspace $V \subset \mathbb{C}^4$ spanned by

$$v_1 = \begin{pmatrix} i \\ 2i \\ 2i \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1+i \\ 0 \\ 1-i \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ i \end{pmatrix}.$$

3. p. 313, problem 2,3.
4. p. 314, problem 30, 32 (b), (c).
5. Find all matrices which are similar to the identity matrix.

Also look at Chris Rasmussen's webpage: http://math.rice.edu/~ crasmus/355/Homework.html