MATH 355 Section 001: Linear Algebra **Fall 2004**

Homework 5, due Friday 10/1

1. Using the theorems from the lecture show that if a square matrix A has two identical rows, then $\det(A) = 0$.

2. Find examples of symmetric 2×2–matrices A, B such that the product AB is not symmetric.

3. Solve the equation system

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

4. Let V be the set of all positive real numbers. For $a, b \in V$ we define $a \oplus b := ab$, i.e. multiplication of real numbers and for $\lambda \in R$ we define $\lambda \odot a := a^\lambda$.

   Show that V is a vector space, i.e. show that axioms $(1)(a), (b), (c), (d)$ and $(1)(a), (b), (c), (d)$ hold. In particular show which element in V is the zero (neutral) element, and which element in V is the inverse of an element $a \in V$.

5. Let V be the set of all functions on $\mathbb{R}$. For two functions $f, g \in V$ we define $f \oplus g$ by $(f \oplus g)(t) := f(t) + g(t)$ and for $f \in V$ and $\lambda \in \mathbb{R}$ we define $\lambda \odot f$ by $(\lambda \odot f)(t) := \lambda f(t)$.

   Show that V is a vector space, i.e. show that axioms $(1)(a), (b), (c), (d)$ and $(1)(a), (b), (c), (d)$ hold. In particular show which element in V is the zero element, and which element in V is the inverse of an element $f \in V$.

6. Using the subspace theorem if necessary determine which of the following are vectorspaces.

   (a) The set of all functions of the form $e^t f(t)$ for some function $f(t)$.

   (b) The set of all 4×3 matrices such that the second row is zero.

   (c) The set of all functions such that $f(1) = 2$.

   (d) The set of all invertible matrices.

   (e) The set of all symmetric matrices.