Homework 9, due Friday 10/29

1. p. 214, problems 11, 13
2. p. 214, problems 15, 17
3. Write down the definition of a vector space three times (handwritten).
4. Write down the definition of a linear transformation three times (handwritten).
5. Let V be $\mathbb{R}^3$ and $W = \mathbb{R}^2$. Then consider the map $\varphi : V \to W$ defined by $\varphi(a, b, c) := (0, a + b + c)$).
   (a) Show that φ is a linear transformation.
   (b) Find the kernel of φ.
   (c) Find the image space of φ.
   (d) Verify the formula $\dim(\mathrm{Ker}(\varphi)) + \dim(\mathrm{Im}(\varphi)) = \dim(V)$.
6. Let V be the vector space of all polynomials of degree less or equal than four. Let $W = \mathbb{R}$. Then consider the map $\varphi : V \to W$ defined by $\varphi(p(t)) = p(2)$. Note that this associates to a polynomial $p(t) \in W$ a number, namely $p(2)$, which is an element of W.
   (a) Show that φ is a linear transformation.
   (b) Find the kernel of φ.
   (c) Find the image space of φ.
   (d) Verify the formula $\dim(\mathrm{Ker}(\varphi)) + \dim(\mathrm{Im}(\varphi)) = \dim(V)$.
7. Let $V = W = \mathbb{R}^2$. Determine which of the following are linear transformations.
   (a) Translation by a non–zero vector.
   (b) Rotation by an angle α.
   (c) Reflection about the x–axis.
   (d) Stretching by a factor of two.
8. (For experts, it does not count towards your homework grade). Give an example of a map $\varphi : \mathbb{R} \to \mathbb{R}$ such that $\varphi(v + w) = \varphi(v) + \varphi(w)$ for all v, w but not necessarily $\varphi(\lambda v) = \lambda\varphi(v)$.