Math 355, Fall 2005 **Exam #1** **Due October 7 at 2pm**

You have 60 minutes for this exam. It is a closed book exam and you are not allowed to use calculators. The exam is due on Friday October 7 at 2pm in class. I will not accept exams afterwards.

This exam consists of 2 sections. Problems in **Part I** are True/False. You do not need to show work for these problems. Solutions to problems in **Part II**, however, must show work to earn full credit. Pledge your exam before turning it in. Good luck!

Part I: For each problem decide whether the claim is true or false. No justification is necessary.

1. If A, B are symmetric 2×2–matrices, then AB is symmetric as well.
2. Let A and B be nonsingular $p \times p$ matrices. Then it follows that
$$(ABA^{-1})^{-1} = AB^{-1}A^{-1}.$$
3. Let A be a square matrix and $l \in \mathbb{R}$. Let $B = lA$. Then $\det(B) = l \det(A)$.
4. If A and B are non–singular square matrices, then
   (a) AB is non–singular,
   (b) $A - B$ is non–singular.
5. Let A, B be two square matrices, then $\det(AB) = \det(BA)$.
6. Let A be a matrix with c columns and r rows. Assume that $c > r$. Then $Ax = 0$ always has infinitely many solutions.

Part II on reverse

Part II:

1. Consider the following augmented matrix, representing a linear system $A\mathbf{x} = \mathbf{b}$, in 3 unknowns:

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & p & q \end{array}\right).$$

   (a) Get this matrix 'as far as possible' into row–echelon form. By 'as far as possible' I mean, try to get the first two columns cleaned up.

   (b) For what values of p and q does $A\mathbf{x} = \mathbf{b}$ have a unique solution? Write down the solution for that case (it will be a vector in terms of p and q).

   (c) For what values of p and q does $A\mathbf{x} = \mathbf{b}$ have no solution?

   (d) For what values of p and q does $A\mathbf{x} = \mathbf{b}$ have infinitely many solutions? Write down the general solution for that case.

2. Find the inverse of the following matrix

$$A = \begin{pmatrix} 3 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 5 \end{pmatrix}.$$

3. Compute the determinant of the following matrix:

$$A = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{pmatrix}$$

4. Let V be the vector space of all functions defined on all of $\mathbb{R}$. Which of the following are subspaces of V? Justify your answer.

   (a) The subset of all functions $f(x)$ such that $f(1) - f(4) = 0$.

   (b) The subset of all functions $f(x)$ such that $e^x f(x) = \sin(x)f(x)$.

   (c) The subset of all polynomials of degree less than 4.

5. Let A be a square matrix. Let b, c be vectors. Assume that $Ax = b$ has NO solution. What can you say about the solution set of $Ax = c$? More precisely:

   (a) Is it possible that $Ax = c$ has a solution?

   (b) If $Ax = c$ has a solution, how many solutions does it have?

   Give a short justification for your answers.