Math 355 **Exam #2** **October 5, 2004**

This exam consists of 2 sections. You do not have to show any work for the problems in **Part I**. Solutions to problems in **Part II**, however, must show work to earn full credit. Pledge your exam before turning it in. Good luck!

Part I: No justification is necessary for the following questions.

1. Let $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.
   (a) What is the dimension of the column space of A?
   (b) What is the dimension of the row space of A?
2. What is the definition of a linear transformation?
3. Let $\varphi : V \to W$ be a linear transformation between finite dimensional vector spaces. What is the formula relating
   - the dimension of the kernel of φ,
   - the dimension of the image space of φ,
   - the dimension of V.
4. Let $\varphi_1, \varphi_2 : V \to W$ be linear transformations. Is the map $\varphi : V \to W$ defined by $\varphi(v) := \varphi_1(v) + \varphi_2(v)$ necessarily a linear transformation?
5. Let $B = \{v_1; v_2; v_3\}$ be an ordered basis for a vector space V. Determine $c_B(v_1), c_B(0)$ and $c_B(v_3 - v_2)$.
6. Let $v_1, \ldots, v_k$ be linearly independent vectors in a k–dimensional vector space V. Is $\{v_1, \ldots, v_k\}$ necessarily a basis vor V?
7. Let $B := \{v_1, v_2\}$ be an ordered basis for V and let $\tilde{B} := \{v_1 + v_2; v_2\}$. What is the matrix A such that $c_B(v) = Ac_{\tilde{B}}(v)$ for all v?

Part II on reverse

Part II:

1. Consider
$$v_1 := \begin{pmatrix}0\\1\\0\end{pmatrix}, v_2 := \begin{pmatrix}0\\1\\2\end{pmatrix}, v_3 := \begin{pmatrix}1\\0\\-2\end{pmatrix}, v_4 := \begin{pmatrix}1\\1\\1\end{pmatrix}.$$
Among $v_1, \ldots, v_4$ find a basis for $\operatorname{span}(v_1, \ldots, v_4)$.

2. Let V be the vector space of all polynomials of degree at most four. Determine whether $f_1 := 1 - t$, $f_2 := 1 + 2t^2$, $f_3 := t + 2t^2$ are linearly independent or not.

3. Let V be the vector space of all polynomials of degree less or equal than two. Let B be the ordered basis $\{1; t; t^2\}$. Consider the linear transformation $\varphi : V \to V$ given by $\varphi(p(t)) = p(2) \cdot (t + 2)$. Which matrix represents φ with respect to the basis B?

4. Let V be the vector space of all polynomials of degree less or equal than 27.
   (a) Show that the map $\varphi : V \to \mathbb{R}$ given by $\varphi(p(t)) := \int_0^1 p(t)\, dt$ is a linear transformation.
   (b) Show that φ is onto, i.e. show that the image space of φ is $\mathbb{R}$.
   (c) Determine the dimension of the kernel of φ.