MATH 355 Section 001: Linear Algebra **Fall 2005**

Homework 10, due Friday 11/11

1. Find the general solution to the equation system
$$\begin{pmatrix} i & 0 & 1 & 2 \\ 0 & 1+i & 2 & 4 \\ 0 & 1 & i & 2+i \end{pmatrix} v = 0$$
where $v \in \mathbb{C}^4$.

2. Find the eigenvalues and the corresponding eigenvectors of
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & 2i \end{pmatrix}.$$

3. Let A be a 4×4–matrix and v_1, v_2, v_3, v_4 a basis for $\mathbb{R}^4$ such that v_1 is an eigenvector for A corresponding to the eigenvalue -2 and v_2 is an eigenvector for A corresponding to the eigenvalue $i-2$. Let $B = \begin{pmatrix} v_1 & v_2 & v3 & v_4 \end{pmatrix}$. What can you say about $B^{-1}AB$?

4. Let $A = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find the eigenvectors corresponding to the eigenvalues.
   (c) Find a matrix B such that $B^{-1}AB$ is a diagonal matrix.
   (d) Find A^{100}.

5. Show that if v is an eigenvector of the matrix A to the eigenvalue λ, then v is an eigenvector of the matrix $A^2 = AA$ to the eigenvalue λ^2.

6. p. 291, problem 1.

7. p. 292, problems 2, 3.

8. p. 293, problem 16. Hint, since $\lambda = -2$ is an eigenvalue, compute $\frac{\det(A-\lambda I)}{\lambda - 2}$ to get the other eigenvalues.

9. Let A be a matrix, show that the constant part of the characteristic polynomial $\det(A - \lambda I)$ is just the determinant of A. (Hint, what happens for $\lambda = 0$?).