Homework 11, due Friday 11/18

1. p. 293, problem 17, 18.


3. p. 303, problem 10 (remember what numbers and properties matrices \( A \) and \( P^{-1}AP \) have in common).

4. Let \( A \) be a \( 3 \times 3 \)-matrix. Assume that a calculation shows that in particular \( \lambda = 1 \) and \( \lambda = 2 + i \) are eigenvalues. Show that \( A \) is diagonalizable.

5. Let \( A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \) with \( a, b, c \in \mathbb{R} \). Show that \( A \) always has two REAL eigenvalues.

6. Let \( h(k) \) denote the number of hares in the year \( k \), and \( r(k) \) the number of rabbits in the year \( k \). Assume that \( h(0) = 2 \) and \( r(0) = 100 \). Assume that the number of foxes and rabbits changes over the years according to the following rule:

\[
\begin{align*}
    r(k+1) &= 10r(k) - 3h(k) \\
    h(k+1) &= 4r(k) + 2h(k).
\end{align*}
\]

What is the number of hares and rabbits after 2 years? After 20 years? Hint, write

\[
\begin{pmatrix} r(k+1) \\ h(k+1) \end{pmatrix} = A \begin{pmatrix} r(k) \\ h(k) \end{pmatrix}
\]

for some matrix \( A \).

7. Let \( A = \begin{pmatrix} 3 & 1 \\ -9 & -3 \end{pmatrix} \). Find a matrix \( P = (v_1 \ v_2) \) such that \( P^{-1}AP \) is of the form

\[
\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}.
\]

Note that \( v_1 \) is an eigenvector.