MATH 355 Section 001: Linear Algebra **Fall 2005**

Homework 11, due Friday 11/18

1. p. 293, problem 17,18.

2. p. 302, problem 1.

3. p. 303, problem 10 (remember what numbers and properties matrices A and $P^{-1}AP$ have in common).

4. Let A be a 3×3–matrix. Assume that a calculation shows that in particular $\lambda = 1$ and $\lambda = 2 + i$ are eigenvalues. Show that A is diagonalizable.

5. Let $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ with $a, b, c \in \mathbb{R}$. Show that A always has two REAL eigenvalues.

6. Let $h(k)$ denote the number of hares in the year k, and $r(k)$ the number of rabbits in the year k. Assume that $h(0) = 2$ and $r(0) = 100$. Assume that the number of foxes and rabbits changes over the years according to the following rule:

$$\begin{aligned} r(k+1) &= 10r(k) - 3h(k) \\ h(k+1) &= 4r(k) + 2h(k). \end{aligned}$$

   What is the number of hares and rabbits after 2 years? After 20 years? Hint, write

$$\begin{pmatrix} r(k+1) \\ h(k+1) \end{pmatrix} = A \begin{pmatrix} r(k) \\ h(k) \end{pmatrix}$$

   for some matrix A.

7. Let $A = \begin{pmatrix} 3 & 1 \\ -9 & -3 \end{pmatrix}$. Find a matrix $P = (v_1\, v_2)$ such that $P^{-1}AP$ is of the form

$$\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}.$$

   Note that v_1 is an eigenvector.