Homework 13 – This homework will NOT be collected

1. p. 224, problems 1,2.

2. Let \( v_1 = (1, 2, 0, 4)^t, v_2 = (0, 1, 0, 1)^t \).
   (a) Show that \( V := \{ v \in \mathbb{R}^4 \text{ orthogonal to } v_1 \text{ and } v_2 \} \) is a subspace of \( \mathbb{R}^4 \).
   (b) Find a basis for \( V \).


4. Let \( v_1 = (1, 2, 0, 2)^t, v_2 = (0, 1, 1, 0)^t, v_3 = (0, 0, 1, 0)^t \). Using Gram-Schmidt find an orthogonal basis for the subspace \( V \) spanned by \( v_1, v_2, v_3 \).

5. Let \( v_1 = (0, 0, 3, 4, 0)^t, v_2 = (0, -1, 1, 0, 2)^t, v_3 = (1, 0, 1, 0, 0)^t \). Using Gram-Schmidt find an orthogonal basis for the subspace \( V \) spanned by \( v_1, v_2, v_3 \).

6. Let \( V \) be the subspace spanned by \( (1, 2, 2)^t \) and \( (-2, 2, -1)^t \).
   (a) Find an orthonormal basis for \( V \).
   (b) Compute the projection of \( e_1, e_2, e_3 \) onto \( V \).
   (c) Find the matrix representing the projection \( \mathbb{R}^3 \to V \) with respect to the standard basis of \( \mathbb{R}^3 \) and the basis for \( V \) you found in (a).

7. We know that if \( v_1, \ldots, v_k \in V \) is an orthonormal basis for \( V \subseteq \mathbb{R}^l \), then given any \( w \in V \) we can write
   \[ w = (v_1 \cdot w)v_1 + \cdots + (v_k \cdot w)v_k. \]

   Put differently, using the inner product we can easily write \( w \) as a linear combination of \( v_1, \ldots, v_k \). Now assume that \( v_1, \ldots, v_k \in V \) is an orthogonal basis for \( V \subseteq \mathbb{R}^l \) (i.e. we no longer know that \( |v_i| = 1 \)). How can we still write \( w \) as a linear combination of \( v_1, \ldots, v_k \) using the inner product?