MATH 355 Section 001: Linear Algebra **Fall 2005**

Homework 13 – This homework will NOT be collected

1. p. 224, problems 1,2.

2. Let $v_1 = (1, 2, 0, 4)^t, v_2 = (0, 1, 0, 1)^t$.

   (a) Show that $V := \{v \in \mathbb{R}^4$ orthogonal to v_1 and $v_2\}$ is a subspace of $\mathbb{R}^4$.

   (b) Find a basis for V.

3. p. 226, problem 16.

4. Let $v_1 = (1, 2, 0, 2)^t, v_2 = (0, 1, 1, 0)^t, v_3 = (0, 0, 1, 0)^t$. Using Gram-Schmidt find an orthogonal basis for the subspace V spanned by v_1, v_2, v_3.

5. Let $v_1 = (0, 0, 3, 4, 0)^t, v_2 = (0, -1, 1, 0, 2)^t, v_3 = (1, 0, 1, 0, 0)^t$. Using Gram-Schmidt find an orthogonal basis for the subspace V spanned by v_1, v_2, v_3.

6. Let V be the subspace spanned by $(1, 2, 2)^t$ and $(-2, 2, -1)^t$.

   (a) Find an orthonormal basis for V.

   (b) Compute the projection of e_1, e_2, e_3 onto V.

   (c) Find the matrix representing the projection $\mathbb{R}^3 \to V$ with respect to the standard basis of $\mathbb{R}^3$ and the basis for V you found in (a).

7. We know that if $v_1, \ldots, v_k \in V$ is an *orthonormal* basis for $V \subset \mathbb{R}^l$, then given any $w \in V$ we can write

   $$w = (v_1 \cdot w)v_1 + \ldots (v_k \cdot w)v_k.$$

   Put differently, using the inner product we can easily write w as a linear combination of $v_1, \ldots, v_k$. Now assume that $v_1, \ldots, v_k \in V$ is an *orthogonal* basis for $V \subset \mathbb{R}^l$ (i.e. we no longer know that $|v_i| = 1$). How can we still write w as a linear combination of $v_1, \ldots, v_k$ using the inner product?