MATH 355 Section 001: Linear Algebra **Fall 2005**

Homework 14 – Solutions

1. Let A be a matrix with $\det(A) = 2 + i$ and let P be an orthogonal matrix. What's the determinant of PAP^t?
   Solution: We know that for an orthogonal matrix $\det(P) = \pm 1$. We also know that $\det(Q^t) = \det(Q)$ for any matrix. So we get $\det(PAP^t) = \det(P)\det(A)\det(P^t) = 1$.

2. (a) Give an example of a normal NON-DIAGONAL 3×3–matrix A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 2 + i$ with algebraic multiplicities 2 and 1.
   Solution: Take $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2+i \end{pmatrix}$, that's a normal matrix with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 2 + i$ with algebraic multiplicities 2 and 1. Of course this matrix diagonal, but now we can compute PDP^{-1} for any orthogonal matrix P, and we still have a normal 3×3–matrix A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 2 + i$ with algebraic multiplicities 2 and 1. If we take almost any such P we get a non–diagonal matrix.

   (b) Now let A be a normal 5×5–matrix A with eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 2 + i\sqrt{2}$ with algebraic multiplicities 2 and 3. What can you say about the Jordan form of A?
   Solution: We know that A is diagonalizable since it's normal, hence the Jordan form is $J = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2+i \end{pmatrix}$.

3. Let A, B be orthogonal matrices. Show that AB and A^{-1} are orthogonal as well.
   Solution: Remember that a matrix P is orthogonal if and only if $P^{-1} = P^t$. Here we have

$$(AB)^t = B^t A^t = B^{-1} A^{-1} = (AB)^{-1},$$

   so AB is orthogonal. (note that we used the usual properties of taking inverses and transposes, and we used that A, B are orthogonal). Similarly one can show that A^{-1} is orthogonal.