

Homework 4, due Friday 9/22

1. p. 152, problem 1 (a) – (f). Ignore the part about Key Theorem 4.13 on rank and solvability, use the methods introduced in class.
2. Solve the following equations system, i.e. determine whether it has a solution, and if it has a solution, find all possible solutions.

$$\begin{array}{rclclcl} x_1 & + & 2x_2 & - & x_3 & + & 3x_4 & = & 3 \\ 2x_1 & + & 4x_2 & - & 2x_3 & + & 5x_4 & = & 5 \\ & & & & x_3 & + & 3x_4 & = & 5 \end{array}$$

3. Solve the following equations system.

$$\begin{array}{rclclcl} & & b & + & 3c & + & 4d & = & 2 \\ & & & & 2c & + & 4d & = & -4 \\ 2a & + & 2b & + & 3c & + & 4d & = & 3 \\ 2a & + & b & & & + & d & = & 6 \\ & & 2b & + & 6c & - & 2d & = & 5 \end{array}$$

4. p. 153, problem 4.
5. Let A be a square matrix and let b, c be vectors. Assume that $Ax = b$ has infinitely many solutions. What can you say about the solution set of $Ax = c$? More precisely:
 - (a) Does $Ax = c$ necessarily have a solution?
 - (b) If $Ax = c$ has a solution, how many solutions does it have?

Give a short justification for your answers.

6. p. 165, problem 2 (a), (b), (c), (d) (Use the definition of the determinant)
7. p. 166, problem 3 (Use the definition of the determinant)
8. Compute the determinant of

$$\begin{pmatrix} 2 & 0 & 5 \\ 3 & 0 & 7 \\ 1 & 1 & 1 \end{pmatrix}$$

- (a) using the first column,
 - (b) using the second column.
 - (c) Compare the results and the amount of work involved.
9. p. 166, problem 8