Homework 4, due Friday 9/22

1. p. 152, problem 1 (a) – (f). Ignore the part about Key Theorem 4.13 on rank and solvability, use the methods introduced in class.

2. Solve the following equations system, i.e. determine whether it has a solution, and if it has a solution, find all possible solutions.

\[
x_1 + 2x_2 - x_3 + 3x_4 = 3 \\
2x_1 + 4x_2 - 2x_3 + 5x_4 = 5 \\
x_3 + 3x_4 = 5
\]

3. Solve the following equations system.

\[
b + 3c + 4d = 2 \\
2c + 4d = -4 \\
2a + 2b + 3c + 4d = 3 \\
2a + b + d = 6 \\
2b + 6c - 2d = 5
\]


5. Let \( A \) be a square matrix and let \( b, c \) be vectors. Assume that \( Ax = b \) has infinitely many solutions. What can you say about the solution set of \( Ax = c \)? More precisely:

(a) Does \( Ax = c \) necessarily have a solution?
(b) If \( Ax = c \) has a solution, how many solutions does it have?

Give a short justification for your answers.

6. p. 165, problem 2 (a), (b), (c), (d) (Use the definition of the determinant)

7. p. 166, problem 3 (Use the definition of the determinant)

8. Compute the determinant of

\[
\begin{pmatrix}
2 & 0 & 5 \\
3 & 0 & 7 \\
1 & 1 & 1
\end{pmatrix}
\]

(a) using the first column,
(b) using the second column.
(c) Compare the results and the amount of work involved.

9. p. 166, problem 8