Homework 5, due Friday 10/7

The first midterm contains all the material up to (and including) this homework set. The midterm will be a take home midterm which I will hand out on Monday and which is due Friday 10/7 at 2pm in class.
If you have any questions, feel free to come to my office hours Monday 3–4, Thursday 2–3, or make an appointment. Note that I will not be in on Thursday 9/29.
You can also go to Landon's office hour Wednesday 3–4 and Thursday 5–6 in Herman Brown 40, or you can also make an appointment with him.

1. Compute the determinant of
$$\begin{pmatrix} 0 & 1 & -1 & 3 \\ 2 & 0 & 0 & 4 \\ 3 & 0 & 7 & 2 \\ 0 & 3 & 5 & -2 \end{pmatrix}$$

2. Using the theorems from the lecture show that if a square matrix A has two identical rows, then $\det(A) = 0$.

3. Let V be the set of all positive real numbers. For $a, b \in V$ we define $a \oplus b := ab$, i.e. multiplication of real numbers and for $\lambda \in R$ we define $\lambda \odot a := a^\lambda$.

   Verify the following three vector space axioms:
$$\begin{aligned} u \oplus (v \oplus w) &= (u \oplus v) \oplus w \\ \alpha \odot (\beta \odot v) &= (\alpha\beta) \odot v \\ (\alpha + \beta) \odot v &= \alpha \odot v \oplus \beta \odot v \end{aligned}$$

   Note that in the $\alpha\beta$ is the usual product of real numbers and $\alpha + \beta$ is the usual sum of real numbers.

   Given $v \in V$, what is the element w such that $v \oplus w =$ "$0''$. (here "$0''$ means the element in V such that $v \oplus 0 = v$ for all v).

4. Let V be the set of all 2×2–matrices. Define $v \oplus w = v + 2w$ (where the right hand side is the usual addition) and $\lambda \odot v = \lambda v$ (i.e. usual scalar multiplication). Is V with the operations $\oplus$ and $\odot$ a vector space?

5. Which of the following is a vector space? Give a short justification for your answer.
   (a) The set of all functions of the form $e^t f(t)$ for some function $f(t)$.
   (b) The set of all 4×3 matrices such that the second row is zero.

(c) The set of all functions such that $f(1) = 2$.

(d) The set of all invertible matrices.

(e) The set of all symmetric matrices.