Homework 7, due Friday 10/21

1. p. 205, problem 6.

2. Let $B := \{v_1; v_2; v_3\}$ be an ordered basis for a vector space V. What is $c_B(0), c_B(-v_2), c_B(v_1 + v_3)$?

3. Let

$$B_1 := \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}, B_2 := \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}.$$

   (a) What is $c_{B_1}(v)$ and $c_{B_2}(v)$ for $v = (v_1\, v_2\, v_3)^t$?.

   (b) What is the base change matrix from B_2 to B_1, i.e. what is the matrix M such that

$$c_{B_1}(v) = M c_{B_2}(v)$$

   for all v?

4. Let $B = \{v_1; v_2; v_3\}$ be an ordered basis. Let $\tilde{B} := \{v_1 + v_2; v_3; v_2\}$. What is the base change matrix from B to $\tilde{B}$, i.e. what is the matrix M such that

$$c_B(v) = M c_{\tilde{B}}(v)$$

   for all v?

5. p. 205, problems 15 (a), (b) and 16 (a), (b).

6. Let $B := \{v_1; v_2; v_3\}$ be an ordered basis for a vector space V.

   (a) Use the principle of isomorphism to show that $v_1 + v_2 - v_3$ and $v_1 + 2v_2 + v_3$ are linearly independent. (Hint: first find $c_B(v_1 + v_2 - v_3)$ and $c_B(v_1 + 2v_2 + v_3)$).

   (b) Using the principle of isomorphism determine whether $v_1 + v_2 - v_3, 2v_1 + 3v_2$ and $v_1 + 2v_2 + v_3$ are linearly independent.

7. Are the polynomials $1 + t + t^3, 1 - 2t + 3t^3, 2 - t + t^3$ linearly independent in the vector space

$$V := \{\text{polynomials of degree less or equal than } 3\}.$$

   Hint: Pick an ordered basis for V and then apply the principle of isomorphism.