Homework 7, due Friday 10/21


2. Let \( B := \{v_1; v_2; v_3\} \) be an ordered basis for a vector space \( V \). What is \( c_B(0), c_B(-v_2), c_B(v_1 + v_3) \)?

3. Let
\[
B_1 := \begin{Bmatrix} (0) & \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{Bmatrix},
B_2 := \begin{Bmatrix} (0) & \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{Bmatrix}.
\]

(a) What is \( c_{B_1}(v) \) and \( c_{B_2}(v) \) for \( v = (v_1 v_2 v_3)^t \)?

(b) What is the base change matrix from \( B_2 \) to \( B_1 \), i.e. what is the matrix \( M \) such that
\[
c_{B_1}(v) = Mc_{B_2}(v)
\]
for all \( v \)?

4. Let \( B = \{v_1; v_2; v_3\} \) be an ordered basis. Let \( \tilde{B} := \{v_1 + v_2; v_3; v_2\} \). What is the base change matrix from \( B \) to \( \tilde{B} \), i.e. what is the matrix \( M \) such that
\[
c_B(v) = MC_{\tilde{B}}(v)
\]
for all \( v \)?

5. p. 205, problems 15 (a), (b) and 16 (a), (b).

6. Let \( B := \{v_1; v_2; v_3\} \) be an ordered basis for a vector space \( V \).

(a) Use the principle of isomorphism to show that \( v_1 + v_2 - v_3 \) and \( v_1 + 2v_2 + v_3 \) are linearly independent. (Hint: first find \( c_B(v_1 + v_2 - v_3) \) and \( c_B(v_1 + 2v_2 + v_3) \)).

(b) Using the principle of isomorphism determine whether \( v_1 + v_2 - v_3, 2v_1 + 3v_2 \) and \( v_1 + 2v_2 + v_3 \) are linearly independent.

7. Are the polynomials \( 1 + t + t^3, 1 - 2t + 3t^3, 2 - t + t^3 \) linearly independent in the vector space
\[
V := \{\text{polynomials of degree less or equal than 3}\}.
\]

Hint: Pick an ordered basis for \( V \) and then apply the principle of isomorphism.