Homework 9, due MONDAY 11/7

1. Let V be the vector space of all differentiable functions on $\mathbb{R}$. Consider $\varphi : V \to V$ given by $\varphi(f(x)) = e^x \cdot f(x)$. This means that to the function $f(x)$ we associate the function $e^x f(x)$.
   (a) Show that φ is a linear transformation.
   (b) Determine the kernel of φ.
   (c) Determine whether φ is invertible. If φ is invertible, what is the inverse.
   (d) Student A says: "let's use matrices to solve (c), just as we did in class". What do you say to this idea?

2. Let V be a vector space with ordered basis $B := \{v_1; v_2\}$ and W a vector space with ordered basis $\tilde{B} := \{w_1, w_2, w_3\}$. Consider the linear transformation φ with $\varphi(v_1) = w_1 - w_2 + w_3$ and $\varphi(v_2) = -w_2 + 2w_3$
   (a) What is $c_B(v_1), c_B(v_2)$?
   (b) What is $c_{\tilde{B}}(w_1), c_{\tilde{B}}(w_2), c_{\tilde{B}}(w_3)$?
   (c) What is $c_{\tilde{B}}(\varphi(v_1)), c_{\tilde{B}}(\varphi(v_2))$?
   (d) What is the matrix representing φ with respect to the ordered bases B and $\tilde{B}$.

3. p. 261, problems 1, 2.

4. Let A be a $p \times q$–matrix. Consider the linear transformation $\varphi : \mathbb{R}^q \to \mathbb{R}^p$ given by $\varphi(v) = Av$. Take $\{e_1; \ldots; e_p\}$ and $\{e_1; \ldots; e_q\}$ as the ordered bases. What is the matrix representing φ with respect to these bases.

5. Let V be the vector space of all polynomials $p(t)$ of degree less or equal than 3 such that $p(2) = 0$. We take the ordered basis $B := \{t - 2; (t - 2)^2; (t - 2)^3\}$. Let W be the vector space of all polynomials $p(t)$ of degree less or equal than 2. For W we take the ordered basis $C := \{1; t; t^2\}$. Now we take the linear transformation $\varphi : V \to W$ which is defined by $\varphi(p(t)) := p'(t)$.
   (a) What is the matrix representing φ with respect to the ordered bases B and C?
   (b) Does φ have an inverse?
   (c) If φ has an inverse, what is it? More precisely, what is the map $\psi : W \to V$ such that $(\psi \circ \varphi)(p(t)) = p(t)$ for any $p(t) \in V$?