Midterm Exam #1
Math 101 – Single Variable Calculus
Summer 2009

Instructions: This is a 2 hour exam. You may not consult any notes or books
during the exam, and no calculators are allowed. Please show your work and
justify your answers.

Name:

Honor Pledge: On my honor, I have neither received nor given any unau-
thorized aid on this exam.

Signature:

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1. Find the largest possible domain for each function. Express this domain as an interval or a union of intervals. (10 points)

(a) \( g(x) = \sqrt{x^2} \)

(b) \( f(x) = (\sqrt{x})^2 \)
2. (10 points)

(a) The line $L$ with slope $m = 2$ passing through $(1, 4)$. Write a linear equation for the line described above. Give your equation in slope-intercept form.

(b) $x^2 + y^2 + 2x + 2y = 2$. Sketch the translated circle described above. Indicate the center and radius of it.
3. Calculate each given limit. Justify each step with the appropriate limit rule. (10 points)

(a) \( \lim_{x \to 0} \frac{\sin 5x}{x} \)

(b) \( \lim_{x \to -2} \frac{x+2}{x^2+4} \)
4. For each function, calculate the derivative using the limit definition of the derivative. (10 points)

(a) \( f(x) = 2x - 1 \)

(b) \( g(x) = \sqrt{2x + 1} \) for \( x > -\frac{1}{2} \)
5. For the curve given below, find all points on the graph where the tangent line is either horizontal or vertical.

\[ y = x^{1/2} - x^{3/2} \]

(15 points)
6. For each function, state where it is differentiable and calculate the derivative. Justify each step using the appropriate derivative law or limit law. (15 points)

(a) \( g(x) = \frac{x^2-4}{x^2+4} \)

(b) \( h(x) = \frac{3x}{x^3+7x-5} \)
7. Let \( f(x) \) and \( g(x) \) be differentiable at \( x \). (15 points)

(a) Show that, for any \( h \neq 0 \) where \( f(x + h) \) and \( g(x + h) \) are defined,
\[
\frac{f(x + h)g(x + h) - f(x)g(x)}{h} = g(x + h)\frac{f(x + h) - f(x)}{h} + f(x)\frac{g(x + h) - g(x)}{h}.
\]

(b) Evaluate the limit
\[
\lim_{h \to 0} \left( g(x + h)\frac{f(x + h) - f(x)}{h} + f(x)\frac{g(x + h) - g(x)}{h} \right).
\]
Write your answer in terms of \( f(x) \), \( g(x) \), \( f'(x) \), and \( g'(x) \).
(c) State the product rule for derivatives and use (a) and (b) to give a proof of it.
8. **Find the maximum and minimum of each function on the given interval.** (15 points)

(a) \( f(x) = 3x - 2 \) on the interval \([-2, 3]\).


(b) \( g(x) = 2x^3 - 9x^2 - 12x \) on the interval \([0, 4]\).