

**Midterm Exam #1**  
**Math 101 – Single Variable Calculus**  
**Summer 2009**

**Instructions:** This is a 2 hour exam. You may not consult any notes or books during the exam, and no calculators are allowed. Please show your work and justify your answers.

**Name:**

**Honor Pledge:** On my honor, I have neither received nor given any unauthorized aid on this exam.

**Signature:**

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
<b>Total</b>	

1. Find the largest possible domain for each function. Express this domain as an interval or a union of intervals. (10 points)

(a)  $g(x) = \sqrt{x^2}$

(b)  $f(x) = (\sqrt{x})^2$

2. (10 points)

- (a) The line  $L$  with slope  $m = 2$  passing through  $(1, 4)$ . Write a linear equation for the line described above. Give your equation in slope-intercept form.

- (b)  $x^2 + y^2 + 2x + 2y = 2$ . Sketch the translated circle described above. Indicate the center and radius of it.

3. Calculate each given limit. Justify each step with the appropriate limit rule. (10 points)

(a)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

(b)  $\lim_{x \rightarrow -2} \frac{x+2}{x^2+4}$

4. For each function, calculate the derivative using the limit definition of the derivative. (10 points)

(a)  $f(x) = 2x - 1$

(b)  $g(x) = \sqrt{2x + 1}$  for  $x > -\frac{1}{2}$

5. For the curve given below, find all points on the graph where the tangent line is either horizontal or vertical.

$$y = x^{1/2} - x^{3/2}$$

(15 points)

6. For each function, state where it is differentiable and calculate the derivative. Justify each step using the appropriate derivative law or limit law. (15 points)

(a)  $g(x) = \frac{x^2-4}{x^2+4}$

(b)  $h(x) = \frac{3x}{x^3+7x-5}$

7. Let  $f(x)$  and  $g(x)$  be differentiable at  $x$ . (15 points)

(a) Show that, for any  $h \neq 0$  where  $f(x+h)$  and  $g(x+h)$  are defined,

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h} = g(x+h)\frac{f(x+h) - f(x)}{h} + f(x)\frac{g(x+h) - g(x)}{h}.$$

(b) Evaluate the limit

$$\lim_{h \rightarrow 0} \left( g(x+h)\frac{f(x+h) - f(x)}{h} + f(x)\frac{g(x+h) - g(x)}{h} \right).$$

Write your answer in terms of  $f(x)$ ,  $g(x)$ ,  $f'(x)$ , and  $g'(x)$ .

- (c) State the product rule for derivatives and use (a) and (b) to give a proof of it.

8. Find the maximum and minimum of each function on the given interval. (15 points)

(a)  $f(x) = 3x - 2$  on the interval  $[-2, 3]$ .

(b)  $g(x) = 2x^3 - 9x^2 - 12x$  on the interval  $[0, 4]$ .