

# Math 102 Spring 2008: Solutions: HW #1

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1. section 7.2, #2

Substitute  $u = 1 + 2x$  so that  $du = 2dx$  and we get

$$\int \frac{1}{u^2} du / 2 = -\frac{1}{2u} + C = -\frac{1}{2(1+2x)} + C.$$

2. section 7.2, #8

Substitute  $u = \pi(2x + 1)$  so that  $du = 2\pi dx$  and we get

$$\int \sin(u) \frac{du}{2\pi} = -\frac{\cos(u)}{2\pi} + C = -\frac{\cos(\pi(2x+1))}{2\pi} + C.$$

3. section 7.2, #16

Substitute  $u = 1 + e^{2x}$  so that  $du = 2e^{2x} dx$  and we get

$$\int \frac{e^{2x}}{u} \frac{du}{2e^{2x}} = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln(1 + e^{2x}) + C.$$

Notice that  $1 + e^{2x} > e^{2x} > 0$  so we can take out the absolute value signs around  $|u|$ .

4. section 7.2, #34

If  $u = x - 1$  then  $x = u + 1$  and  $du = dx$  and we get

$$\begin{aligned} \int (u+1)u^{1/3} du &= \int u^{4/3} + u^{1/3} du \\ &= \frac{3}{7}u^{7/3} + \frac{3}{4}u^{4/3} + C \\ &= \frac{3}{7}(x-1)^{7/3} + \frac{3}{4}(x-1)^{4/3} + C \end{aligned}$$

5. section 7.2, #52

We substitute  $u = x + 2$  so  $du = dx$ . We get

$$\begin{aligned} \int \frac{1}{(x+2)^2+1} dx &= \int \frac{1}{u^2+1} du \\ &= \tan^{-1}(u) + C \\ &= \tan^{-1}(x+2) + C. \end{aligned}$$

6. section 7.3, #2

Using integration by parts we have

$$\begin{aligned}\int x^2 e^{2x} dx &= x^2 e^{2x} / 2 - \int (2x) e^{2x} / 2 dx \\ &= x^2 e^{2x} / 2 - \int x e^{2x} dx \\ &= x^2 e^{2x} / 2 - (x e^{2x} / 2 - \int e^{2x} / 2 dx) \\ &= x^2 e^{2x} / 2 - x e^{2x} / 2 + e^{2x} / 4 + C.\end{aligned}$$

7. section 7.3, #20

Using integration by parts we have

$$\begin{aligned}\int \sin(\ln(t)) dt &= t \sin(\ln t) - \int t \cos(\ln t) \frac{1}{t} dt \\ &= t \sin(\ln t) - \int \cos(\ln t) dt \\ &= t \sin(\ln t) - (t \cos(\ln t) - \int t(-\sin(\ln t)) \frac{1}{t} dt) \\ &= t \sin(\ln t) - t \cos(\ln t) - \int \sin(\ln t) dt\end{aligned}$$

Thus we get

$$\int \sin(\ln(t)) dt = \frac{1}{2}(t \sin(\ln t) - t \cos(\ln t)).$$

8. section 7.3, #38

Substitute  $t = x^{3/2}$  so that  $x = t^{2/3}$  and  $dt = 3/2 x^{1/2} dx$  and we get

$$\begin{aligned}\int x^2 \sin x^{3/2} dx &= \int x^2 \sin(t) \frac{2/3^{-1/2}}{x} dt \\ &= \int \frac{2}{3} x^{3/2} \sin(t) dt \\ &= \int \frac{2}{3} t \sin(t) dt \\ &= \frac{2}{3} (-t \cos(t) - \int (-\cos(t)) dt) \\ &= \frac{2}{3} (-t \cos(t) + \sin(t)) + C \\ &= \frac{2}{3} (-x^{3/2} \cos(x^{3/2}) + \sin(x^{3/2})) + C\end{aligned}$$

where we also used integration by parts.

9. section 7.3, #50

We use  $u = x^{n-1}$  and  $v' = xe^{-x^2}$ . Then  $v = -\frac{1}{2}e^{-x^2}$  and we get

$$\int x^n e^{-x^2} dx = x^{n-1} \left(-\frac{1}{2}e^{-x^2}\right) - \int (n-1)x^{n-2} \left(-\frac{1}{2}e^{-x^2}\right) dx = -\frac{1}{2}x^{n-1}e^{-x^2} + \frac{n-1}{2} \int x^{n-2} e^{-x^2} dx$$

which is what we wanted to show.

10. section 7.3, #56

Using the recursion from problem 50 repeatedly we get

$$\begin{aligned} \int_0^1 x^5 e^{-x^2} dx &= -\frac{1}{2}x^4 e^{-x^2} \Big|_0^1 + \frac{4}{2} \int x^3 e^{-x^2} dx \\ &= -\frac{1}{2}e^{-1} + 2\left(-\frac{1}{2}x^2 e^{-x^2} \Big|_0^1 + \frac{2}{2} \int x^1 e^{-x^2} dx\right) \\ &= -\frac{1}{2e} - e^{-1} + 2(e^{-x^2}/(-2)) \Big|_0^1 \\ &= -\frac{1}{2e} - \frac{1}{e} - (e^{-1} - e^0) \\ &= -\frac{3}{2e} - \frac{1}{e} + 1 \\ &= -\frac{5}{2e} + 1. \end{aligned}$$