Math 102 Spring 2008: **Solutions: HW #4**  
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1. section 10.2, #6  
The denominators are always one more than a square. So the $n$th denominator is $n^2 + 1$ so the general term in the sequence is  
   
   $$a_n = \frac{1}{n^2 + 1}.$$  

2. section 10.2, #12  
Dividing top and bottom by $n^2$ we get $\frac{n}{10 + 1/n}$. Now $10 + 1/n \to 10$ as $n \to \infty$ so we get that $\frac{n}{10 + 1/n}$ must diverge.

3. section 10.2, #14  
We have $(-1/2)^n = (-1)^n \frac{1}{2^n} \to 0$ as $n \to \infty$. Thus  
   
   $$\lim_{n \to \infty} 2 - (-1/2)^n = 2.$$  

4. section 10.2, #20  
We have $-1 \leq \cos n \leq 1$ so $1 \leq 2 + \cos n \leq 3$ and hence  
   
   $$\frac{1}{n} \leq \frac{2 + \cos n}{n} \leq \frac{3}{n}.$$  

Thus  
   
   $$\frac{1}{\sqrt{n}} \leq \sqrt{\frac{2 + \cos n}{n}} \leq \frac{3}{\sqrt{n}}.$$  

Since $\frac{1}{\sqrt{n}} \to 0$ this means that  
   
   $$\sqrt{\frac{2 + \cos n}{n}} \to 0$$  

as $n \to \infty$.

5. section 10.2, #30  
Using L’hôpital’s rule four times we have  
   
   $$\lim_{x \to \infty} \frac{x^3}{e^{x/10}} = \lim_{x \to \infty} \frac{3x^2}{e^{x/10}/10} = \lim_{x \to \infty} \frac{6x}{e^{x/10}/100} = \lim_{x \to \infty} \frac{6}{e^{x/10}/1000} = 0$$  

This means that the sequence converges and the limit is zero.
6. section 10.2, #34
Let’s look at $b_n = \ln(a_n) = \frac{1}{n}\ln(2n + 5)$. Now by L’hôpital’s rule
\[
\lim_{x \to \infty} \frac{\ln(2x + 5)}{x} = \lim_{x \to \infty} \frac{2/(2x + 5)}{1} = 0
\]
So $b_n \to 0$ which means $a_n \to 1$.

7. section 10.2, #50
We have
\[
\lim_{n \to \infty} \frac{3n - 1}{4n + 1} = \lim_{n \to \infty} \frac{3 - 1/n}{4 + 1/n} = \frac{3}{4}
\]
where we divided the top and bottom by $n$ to get the first equality. Thus
\[
\lim_{n \to \infty} 3 \sin^{-1} \left( \frac{3n - 1}{4n + 1} \right) = 3 \sin^{-1} \left( \lim_{n \to \infty} \frac{3n - 1}{4n + 1} \right) = 3 \sin^{-1} \left( \frac{\sqrt{3}}{4} \right) = \pi.
\]

8. section 10.2, #56
Since $a_{n+1} = 1 + (1/a_n)$ and the limit $\lim_{n \to \infty} a_n = L$ exists we can take the limit of both sides to get
\[
L = 1 + 1/L.
\]
This gives $L^2 - L - 1 = 0$ which means (by the quadratic formula) that
\[
L = \frac{1 \pm \sqrt{5}}{2}
\]
Since $L$ is clearly non-negative this means
\[
L = \frac{1 + \sqrt{5}}{2}.
\]