Math 212 Spring 2008 Exam 2

Instructor: S. Cautis

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **two hours**. Do all 8 problems. You must show your work to receive full credit on a problem. An answer with no supporting work or explanation will receive little to no credit.
Please do all your work on stardard letter size sheets. At the end write your name on each sheet, attach this page to the front and staple!
The exam is due at the **beginning of class on April 10.**

Please print you name clearly here.

Print name: ________________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam and have observed the time limit given. I started working on this exam at ___:___ and finished at ___:___ on the ___th day of April.

____

Grader's use only:

1. ________ /20

2. ________ /15

3. ________ /25

4. ________ /20

5. ________ /20

6. ________ /15

7. ________ /15

8. ________ /20

1. [20 points]

   Find the extreme values of

   $$f(x, y) = x^2 + 3y^2 - 4x + 3$$

   over the region bounded by the curves $y^2 = x$ and $x = 4$.

2. a)[5 points] Sketch the vector field $\mathbf{F}(x, y) = (-y, x)$ at the eight points $(1, 0)$, $(1, 1)$, $(0, 1)$, $(-1, 1)$, $(-1, 0)$, $(-1, -1)$, $(0, -1)$ and $(1, -1)$.

   b)[5 points] Sketch the flow line through the point $(1, 0)$.

   c)[5 points] Find an explicit parametrization of the flow line through the point $(1, 0)$ and check that it is indeed a flow line.

3. a)[10 points] If $\mathbf{F}(x, y, z)$ is a vector field prove that $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

   b)[5+5 points] Compute the curl and divergence of $\mathbf{F}(x, y, z) = (-xy, xz, -yz)$.

   c)[5 points] Can $\mathbf{F}(x, y, z) = (-xy, xz, -yz)$ be the curl of a vector field? (i.e. does there exist a vector field $\mathbf{V}$ such that $\nabla \times \mathbf{V} = \mathbf{F}$.) If yes then find such a $\mathbf{V}$ – otherwise justify why not.

4. a)[10 points] Consider the triangle T with vertices $(1, 1)$, $(3, 2)$ and $(2, 4)$ and the triangle T' with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. Find a change of variables $f : \mathbf{R}^2 \to \mathbf{R}^2$ which maps the triangle T' onto the triangle T.

   b)[10 points] Let E be the ellipse given by $x^2 + 4y^2 = 4$. Find a change of coordinates $f : \mathbf{R}^2 \to \mathbf{R}^2$ which maps the square $[0, 1] \times [0, 1]$ onto the (interior of the) ellipse E.

5. [20 points] Compute the integral $\int_0^2 \int_0^{x^2} (y + 3x^2) dydx$ by **first** changing the order of integration and **then** integrating.

6. [15 points]

   Let D be the region in $\mathbf{R}^2$ determined by conditions $4x^2 + 9y^2 \leq 36$ and $x, y \geq 0$. Compute the integral

   $$\int\int_D y dx dy$$

7. Let $f(x, y) = e^{x^2+y} - 1$.
   a)[6 points] Find the second order Taylor series expansion of $f(x, y)$ around the point $(0, 0)$.
   b)[6 points] Find the second order Taylor series expansion of $f(x, y)$ around the point $(0, 1)$.
   c)[3 points] Give an example of a smooth function $g(t)$ whose Taylor series around 0 does **not** converge to $g(t)$?

8. [20 points]

   Let W be the region in $\mathbf{R}^3$ determined by conditions $x^2 + y^2 + z^2 \leq 1$ and $z^2 \geq x^2 + y^2$. Compute the volume of W. (hint: it may be helpful for you as well as the grader if you drew a picture of W). You should be able to compute the integral and get a number at the end.