Math 212 Spring 2006 **Exam #2**

Instructions: You have **2 hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 6 questions. You must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question. Pledge your exam when finished, and include your name and section number on the front of the exam. The exam is due by **Wednesday, 4 p.m.** Good luck!

1. Find and classify all the critical points of
$$f(x,y) = \frac{1}{2}x^2 - xy + \frac{1}{3}y^3.$$

2. Let $g(x,y) = 2e^{-x}\cos y$.
   (a) Find the quadratic Taylor polynomial for $g(x,y)$ around the point $(0,0)$.
   (b) Use your answer in part (a) to estimate $2e^{-0.2}\cos 0.4$.

3. A tank is in the shape of a half-cylinder of radius 2 and height 3. It is situated in $\mathbb{R}^3$, given by the inequalities $\sqrt{x^2+y^2} \le 2$, $y \ge 0$, and $0 \le z \le 3$. The temperature at the point (x,y,z) is given by
$$T(x,y,z) = 2yz^2\sqrt{x^2+y^2} \ \ ^\circ\text{C}.$$
Find the average temperature in the tank.

4. Let T be the triangle with vertices $(0,0), (1,1)$ and $(0,1)$ and let $f(x,y) = x\sin(y^3)$.
   (a) Find the correct limits of integration to **set up** $\displaystyle\iint_T f(x,y)\,dA$ as a double integral $\displaystyle\iint f(x,y)\,dx\,dy$.
   (b) Find the correct limits of integration to **set up** $\displaystyle\iint_T f(x,y)\,dA$ as a double integral $\displaystyle\iint f(x,y)\,dy\,dx$.
   (c) Compute $\displaystyle\iint_T f(x,y)\,dA$.

5. Find the maximum and minimum values obtained by $f(x,y) = x + y^2$ on the ellipse $x^2 + 3y^2 \le 9$.

6. The region S is cut from a solid ball of radius 1 centered at the origin. S is the region cut by the inequalities $z \ge 0$ and $y \ge x$. (S is one-quarter of the entire ball, and contains the point $(0,1,0)$.)
   The mass density of S at a point (x,y,z) is given by the function $\delta(x,y,z) = 30z^2$ kg/m^3.
   (a) Find the total mass of S.
   (b) Find the average mass density of S.