

Homework 4, due Wed.Sept.26

Exercise 2.70, Exercise 2.71 (include pictures in your write-up of these 2 problems)

As in the hints, you may use the fact that a continuous function on a compact (closed and bounded) subset of Euclidean space attains a minimum and maximum.

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Fifth Exercise: Suppose that  $p = (p_1, p_2, p_3)$  is a point on the torus of revolution  $S$  of Example 2.17 (page 37). Find a formula describing the general point  $(x, y, z)$  in the tangent plane  $T_p S$ , in terms of  $p_1, p_2, p_3, a, r$ .

Last exercise (Taylor Series in terms of curvature and torsion): Suppose  $\alpha(s)$  is a smooth curve parameterized by arc-length with positive curvature  $\kappa(s)$  and torsion  $\tau(s)$ . Suppose also that  $\alpha(0) = (0, 0, 0)$ ,  $\alpha'(0) = (1, 0, 0)$  and  $\alpha''(0) = \kappa(0)(0, 1, 0)$ . Find formulas for  $\alpha'(s), \alpha''(s), \alpha'''(s)$  and  $\alpha''''(s)$  in terms of the tangent, normal, and binormal  $T(s), N(s), B(s)$  as well as  $\kappa(s)$  and  $\tau(s)$  and their derivatives. By evaluating these at  $s = 0$ , find the degree 3 Taylor approximation of  $\alpha(s)$  at 0. That is, find the vectors  $C_0, C_1, C_2, C_3$  so that

$$\alpha(s) = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + \text{higher order terms}.$$